# Physics

## B. Sc. III – Semester

# Study Material

(As per the syllabus of Adikavi Nannaya University, Rajamahendravaram)

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## Chapter - I
# Aberrations

**Aberration:-**

The error in shape or colour in the formation of the image by an optical instrument is called aberration.

Aberrations are of two types.

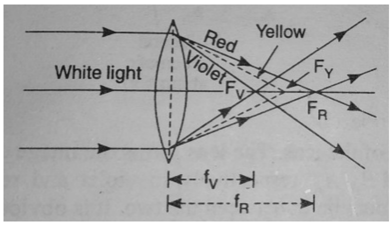
1. Aberration due to the light used.
2. Aberration due to the optical instrument used.

**I. The aberration due to the light or chromatic aberration** :-

**Definition** :- If a white or composite light incident on an optical instrument and if the image formed is in different colours, that aberration is called chromatic aberration.

- This aberration arises due to the colour of the light.
- This aberration is observed only in lenses but not in mirrors.
- The refractive index changes with the wavelength of the light. Different colours will have different refractive indices.
  As per lens maker's formula

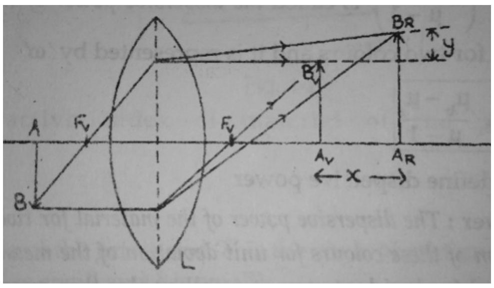
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$




- ✓ Here the focal length (f) depends on the refractive index (μ) of the meterial of the lens. So, the focal length varies with colour Here $R_1$ , $R_2$ are the radii of curvatures of the two surfaces of the lens.
- ✓ Here the focal length (f) is inversely proportional to the refractive index (μ).
- ✓ Since $\mu_V > \mu_R$ then , $f_R > f_V$.
- ✓ So, different images with different colours will be formed at different distances. This is chromatic aberration.

Chromatic aberration is of two types.

1. Longitudinal (OR) Axial chromatic aberration
2. Lateral (or) transverse chromatic aberration




**Longitudinal chromatic aberration** :- The variation of the image distance from the lens with colour is called axial or longitudinal chromatic aberration.

It is shown as $A_VA_R = x$ in the figure.

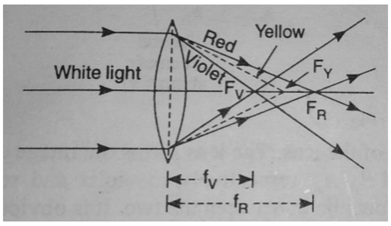
**Lateral (or) transverse chromatic aberration** :- The change in the size of the image with colour is called lateral are transverse chromatic aberration.

It is shown as $A_RB_R - A_VB_V = y$ in the figure.

**<u>Derivation of equation for chromatic aberration</u>**

**<u>case 1 :-</u>** <u>when object is placed at infinite distance</u>

Take a convex lens of μ refractive index and f focal length. Let the radii of curvature of the two surfaces of a lens are ,$_1$RR.$_2$ Let a white object is placed at Infinity distance from the lens and the parallel beam of light rays incident on the lens.




The lens maker's formula is $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$

( (OR $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f(\mu-1)}$ ⟶ (1)

Equation for violet colour is $\frac{1}{f_V} = (\mu_V - 1)(\frac{1}{R_1} - \frac{1}{R_2})$ ⟶ (2)

Equation for red colour is $\frac{1}{f_R} = (\mu_R - 1)(\frac{1}{R_1} - \frac{1}{R_2})$ ⟶ (3)

Subtracting equation (3) from equation (2)

$$\frac{1}{f_V} - \frac{1}{f_R} = (\mu_V - \mu_R)(\frac{1}{R_1} - \frac{1}{R_2}) \longrightarrow (4)$$

Substituting equation (1) in equation (4)

$$\frac{1}{f_V} - \frac{1}{f_R} = \frac{1}{f}\frac{(\mu_V-\mu_R)}{(\mu-1)}$$

$$\frac{f_R-f_V}{f_Rf_V} = \frac{1}{f}\frac{(\mu_V-\mu_R)}{(\mu-1)}$$

$$f_Rf_V = f^2$$

Here f is the average focal length.

$\therefore \frac{f_R-f_V}{f^2} = \frac{1}{f}\frac{(\mu_V-\mu_R)}{(\mu-1)}$ ( or) $\frac{f_R-f_V}{f} = \frac{(\mu_V-\mu_R)}{(\mu-1)}$

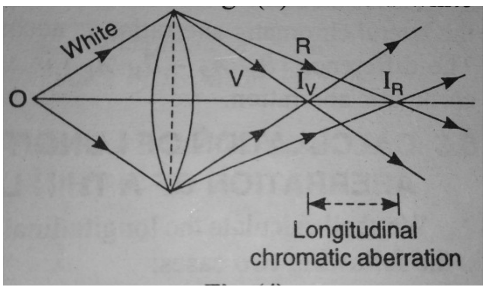
But $\frac{(\mu_V-\mu_R)}{(\mu-1)} = \omega$ Here $\omega$is the dispersive power of the lens .

$\frac{f_R-f_V}{f} = \omega$ ( or) $\boxed{f_R - f_V = \omega f}$

when the object is placed at infinite distance, then the longitudinal chromatic aberration is equal to the difference in the focal lengths of the two colours.

**<u>Case 2</u>** :- <u>When the object is placed at finite distance</u>

Consider a lens of focal length f and dispersive power ω. Let a white object is placed at an distance u from the lens on the axis and its images formed at a distance v on the other side of the lens.




The focal length of the lens is

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ ⟶ (1)

f and V values change with colour but u value is a constant.

Differentiating equation (1) $-\frac{df}{f^2} = -\frac{dv}{v^2} - 0$

$$\frac{df}{f^2} = \frac{dv}{v^2}$$

$df = f_R - f_V$ = change in the focal length

$dv = v_R - v_V$ = change in the image distance

$\therefore \frac{f_R - f_V}{f^2} = \frac{v_R - v_V}{v^2}$ (or( $v_R - v_V = \frac{f_R - f_V}{f^2} . v^2$ ⟶ (2)

But, $f_R - f_V = \omega f$ ⟶ (3)

Substituting equation (3) in equation (2)

$$v_R - v_V = \frac{\omega f}{f^2} . v^2$$

(or( $$\boxed{v_R - v_V = \frac{\omega}{f} . v^2}$$

It means that the object is placed at finite distance then longitudinal aberration is equal to the difference between the image distances of different colours.

**Achromatism**

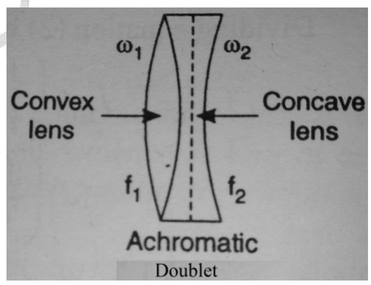
Definition :- The process of minimization or removal of chromatic aberration is called achromatism.

Explanation :- Chromatic aberration is positive to the convex lens and it is negative to the concave lens. Hence to eliminate or minimize the chromatic aberration a combination of a convex and a concave lenses can be used. This method is called achromatism.

Achromatism can be obtained in two methods.

I. Achromatic doublet 2) Achromatic combination

**Achromatic doublet**




- The combination of a convex lens and a concave lens placed in contact to eliminate or minimize the chromatic aberration is called " achromatic doublet."
- This is used in microscopes and telescopes as objective.
- Achromatic doublet converges all the rays of composite light at one point. So, all the colours have the same focal length and it does not depend on refractive index.

Lens maker's formula is $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$ ⟶ (1)

Here $f$ = Focal length of the lens

$\mu$ = Refractive index of the material of the lens

$R_1, R_2$ are radii of curvature of the two surfaces of the lens

renciating eqnDiffe. (1) $d\left(\frac{1}{f}\right) = d\mu.(\frac{1}{R_1} - \frac{1}{R_2})$ ⟶ (2)

Dividing eqn. (2) by eqn. (1)

$$f.d\left(\frac{1}{f}\right) = \frac{d\mu}{(\mu-1)}$$

$$f.d\left(\frac{1}{f}\right) = \omega$$

$\because \frac{d\mu}{(\mu-1)} = \omega = \textit{Dispersive power of te material of the lens}$

( or ) $\boxed{d\left(\frac{1}{f}\right) = \frac{\omega}{f}}$ ⟶ (3)

Let $f_1$, $f_2$ be the focal lengths of the two lenses in the achromatic doublet and the dispersive powers of those materials are $\omega_1$, $\omega_2$. Let F be the focal length of the combination.

Then $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ ⟶ (4)

✓ The two lenses have chromatic aberration when they are individual. So, different colour have different focal lengths. So, $f_1, f_2$ are variables.

- ✓ But the achromatic doublet should have one focal length. So, F is constant.

Differentiating eqn. (4) $d(\frac{1}{F}) = d(\frac{1}{f_1}) + d(\frac{1}{f_2})$

$$0 = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} \quad \text{as per eqn. (3)}$$

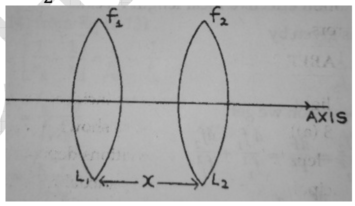
( (or $\boxed{\frac{f_1}{f_2} = -\frac{\omega_1}{\omega_2}}$

So, the chromatic aberration can be eliminated if the ratio of the focal lengths is equal to the ratio of their dispersive powers. As $\omega_1, \omega_2$ both are positives, one of $f_1, f_2$ should be positive and the other should be negative. It means one lens is convex and the other is concave.

**<u>Achromatic combination (or) Separated achromatic doublet</u>**

If two lenses are placed coaxially with some separation, to eliminate chromatic aberration, is called **achromatic combination** (or) **separated achromatic doublet**.

Lens maker's formula for one lens is $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2}) \longrightarrow (1)$




Here $f$ = Focal length of the lens

$\mu$ = Refractive index of the material of the lens

$R_1, R_2$ are radii of curvature of the two surfaces of the lens

differentiating equation (1)

$$d\left(\frac{1}{f}\right) = d\mu.(\frac{1}{R_1} - \frac{1}{R_2}) \longrightarrow (2)$$

Dividing equation (2) by equation (1)

$$f.d\left(\frac{1}{f}\right) = \frac{d\mu}{(\mu-1)}$$

$f.d\left(\frac{1}{f}\right) = \omega \qquad \because \frac{d\mu}{(\mu-1)} = \omega =$ Dispersive power of the material of the lens

(or) $\boxed{d\left(\frac{1}{f}\right) = \frac{\omega}{f}} \longrightarrow (3)$

Let $f_1$ and $f_2$ be the focal lengths of the two lenses in the combination and the dispersive powers of the materials are $\omega_1$, $\omega_2$. Let F be the focal length of the combination and the distance between the two lenses is x.

Then $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} \longrightarrow (4)$

- ✓ The two lenses have chromatic aberration when they are individual. So, different colour have different focal lengths. So, $f_1$, $f_2$ are variables.
- ✓ But the achromatic combination should have one focal length. So, F is constant.

Here x is also a constant.

Differentiating equation (4)

$$d\left(\frac{1}{F}\right) = d\left(\frac{1}{f_1}\right) + d\left(\frac{1}{f_2}\right) - \frac{x}{f_2}.d\left(\frac{1}{f_1}\right) - \frac{x}{f_1}.d\left(\frac{1}{f_2}\right)$$

$$0 = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{x}{f_2}.\frac{\omega_1}{f_1} - \frac{x}{f_1}.\frac{\omega_2}{f_2} \quad \text{as per equation (3)}$$

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \frac{x}{f_2}.\frac{\omega_1}{f_1} + \frac{x}{f_1}.\frac{\omega_2}{f_2}$$

$$\frac{\omega_1 f_2+\omega_2 f_1}{f_1 f_2} = \frac{x(\omega_1+\omega_2)}{f_1 f_2}$$

(or( $$\boxed{x = \frac{(\omega_1 f_2+\omega_2 f_1)}{(\omega_1+\omega_2)}}$$

If the two lenses are made with the same material. Then $\omega_1 = \omega_2 = \omega$

And $x = \frac{(\omega f_2+\omega f_1)}{(\omega+\omega)} = \frac{\omega(f_2+f_1)}{(2\,\omega)}$

$$\boxed{\therefore\ x = \frac{(f_2+f_1)}{2}}$$

The chromatic aberration can be eliminated, if two lenses are placed coaxially at a distance equal to the average focal length of the two lenses and the two lenses are made with the same material.

**II. <u>Aberrations due to the optical instrument</u>**

**<u>Monochromatic aberration</u>** :- The defect is observed in the image even though the light used is a monochromatic light that aberration is called mono chromatic aberration.

- This aberration is due to the lens or the mirror used in the optical instrument. Monochromatic aberrations are of different types. They are

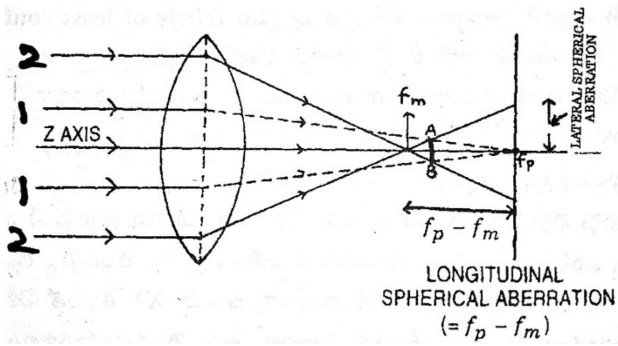
1. Spherical aberration
2. Coma
3. Astigmatism
4. Curvature of field
5. Distortion

**<u>Spherical aberration</u>** :- The inability of the lens or mirror to form a point image at a single point on the axis, of a point object of single colour placed on the axis of the lens or mirror is called Spherical aberration.

- ✓ Spherical aberration arises due to the lenses and also due to the mirrors.

**<u>Reason for Spherical aberration</u>** :-

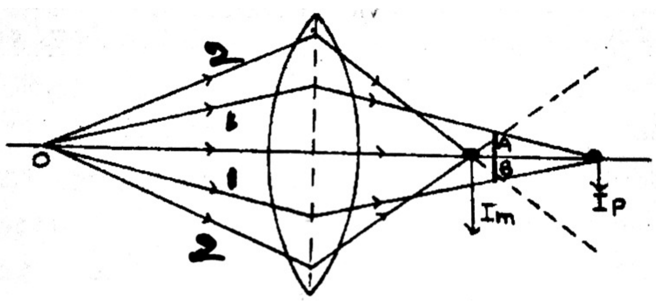
Consider a convex lens as shown in the figure. The light rays (2, 2) are incidenting on the lens away from the principal axis. These rays are called marginal rays. Similarly the light rays (1, 1) are incidenting on the lens very close to the principal axis and these rays are called paraxial rays.




The angular deviation (δ) produced in a ray is directly proportional to the height (h) of the incident ray from the axis of the lens $\delta = \frac{h}{f}$

f = Focal length of the lens

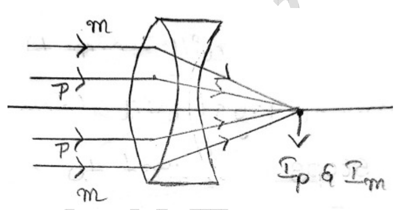
- So, the paraxial rays (1, 1) incident in on the lens very close to the axis have less h & the deviation (δ) is also less.



- But the marginal rays (2, 2) incident far away from the axis of a lens have large h value and the deviation (δ) is also large.
- The Paraxial rays form the image at $f_p$ and marginal rays form the image at $f_m$.
- $f_p$ is called paraxial focus and $f_m$ is called the marginal focus so paraxial and marginal rays have different focal lengths.

**<u>Elimination or minimization of Spherical aberration</u>** :-

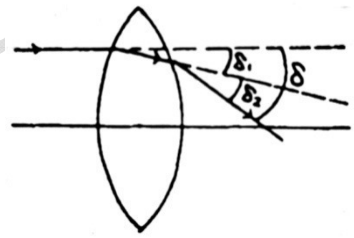
1. **<u>By using stops</u>** :- The main reason for spherical aberration is that the paraxial rays and the marginal rays have different focal lengths. So if marginal rays are eliminated by using stops then the paraxial rays will only form the image without having any spherical aberration.
2. **<u>By using two lenses in contact</u>** :- We know that the spherical aberration for convex lens is positive, whereas for concave lens it is negative. The Spherical aberration can be eliminated if the combination of convex and concave lenses are put in contact. That means the marginal and paraxial rays will form the image at one place.




3. **<u>Dividing the deviation equally at the two surfaces of Plano convex lens</u>** :- Spherical aberration is directly proportional to the square of the deviation produced in a ray.

Spherical aberration $\propto \delta^2$

Let $\delta_1$, $\delta_2$ be the deviations produced at the two surfaces of a lens.




Total deviation $\delta = (\delta_1 + \delta_2)$

Spherical aberration $= (\delta_1 + \delta_2)^2$

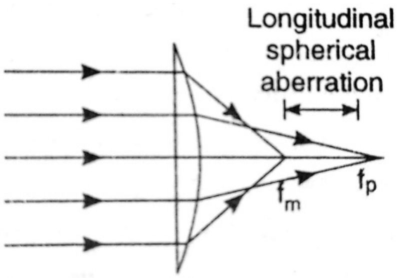
$= (\delta_1 - \delta_2)^2 + 4\,\delta_1\,\delta_2$

The spherical aberration will be minimum, only when $\delta_1 = \delta_2$

The Spherical aberration will become minimum if the deviations produced at the two surfaces of lens are equal.

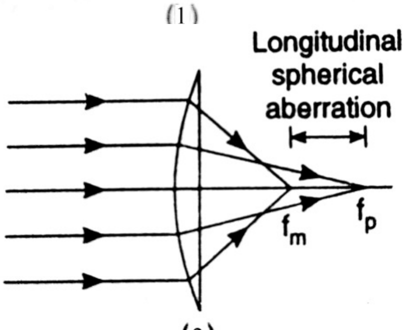
Take a Plano convex lens.

- In <u>figure -1</u> the plane surface of the plano-convex lens is placed towards the object. Here no deviation is produced at the plane surface and the total deviation occurs at the convex surface only. So, the Spherical aberration will become maximum.



(1)

- <u>In figure-2</u> the convex surface of the Plano convex lens is placed towards the object. Here the deviation is produced at the two surfaces of the lens equally. So, the Spherical aberration will become minimum.
- So the Spherical aberration will be minimized by using a Plano convex lens such that the deviations produced at the two surfaces are equal

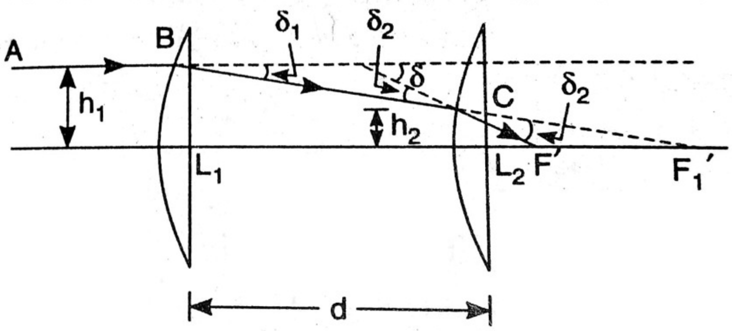


(2)

**<u>Note</u>** :- If the object is placed at finite distance from the lens, then the plane surface of the Plano convex lens should be placed towards the object.

4. **<u>when two Plano convex lenses are separated by some distance</u>** :-

- ✪ Take two Plano convex lenses $L_1$ and $L_2$ as shown in the figure. These two lenses are separated by a distance d. Their focal lengths are $f_1$ and $f_2$ respectively.




- ✪ The light ray AB incident on the lens $L_1$ at a height $h_1$ from its axis and takes a deviation $\delta_1$ and forms the image at $F_1'$.
- ✪ The same light ray incident on the lens $L_2$ at a height $h_2$ from its axis and takes deviation $\delta_2$ and forms the final image at F.
- ✪ The spherical aberration will be minimum, only when $\delta_1 = \delta_2$
- ✪

$$\delta = \frac{h}{f} \qquad \text{So, } \delta_1 = \frac{h_1}{f_1} \text{ and } \delta_2 = \frac{h_2}{f_2}$$

$$\delta_1 = \delta_2 \qquad \therefore \frac{h_1}{f_1} = \frac{h_2}{f_2} \text{ (or) } \frac{h_1}{h_2} = \frac{f_1}{f_2} \longrightarrow (1)$$

From the two similar triangles $BL_1F_1'$ and $CL_2F_1'$

$$\frac{BL_1}{CL_2} = \frac{L_1F_1'}{L_2F_1'} \text{ (or) } \frac{h_1}{h_2} = \frac{f_1}{(f_1 - d)} \longrightarrow (2)$$

From equations (1) and (2)

$$\frac{f_1}{f_2} = \frac{f_1}{(f_1 - d)} \text{ (or) } f_2 = (f_1 - d)$$

$$\therefore \boxed{d = (f_1 - f_2)}$$

The spherical aberration will be minimized when two Plano convex lenses are separated coaxially at a distance equal to the difference of their focal length.

5. <u>Minimization of spherical aberration by using crossed lens</u> :-

- ➢ The convex lens having two surfaces of different radii of curvature is called crossed lens. The ratio of the radii of curvature of the two surfaces is called shape factor (β).
- ➢ If the shape factor (**β**) value and the refractive index (μ) value satisfies the below equation, then the spherical aberration is completely eliminated.

$$\boxed{\beta = \frac{R_1}{R_2} = \frac{\mu(2\mu - 1) - 4}{\mu(2\mu + 1)}}$$

Here $R_1$, $R_2$ are the radii of curvature of the two surfaces of the lens.

If $\beta = \frac{R_1}{R_2} = -\frac{1}{6}$ then spherical aberration is minimized.

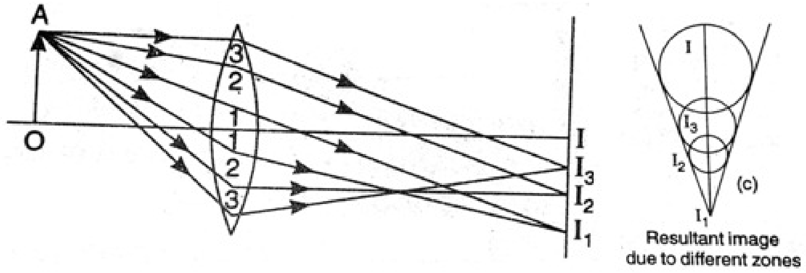
If μ = 1.686 then $\beta = \frac{R_1}{R_2} = 0$ So, $R_2 = \infty$

It means the lens is a plano-convex lens.

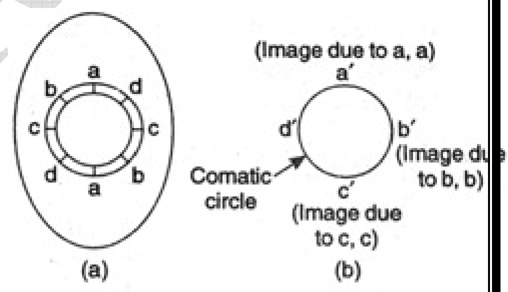
So, a plano- convex lens withμ = 1.686 works as crossed lens .

## Coma

If a monochromatic object of point size is placed slightly off axis of the lens then the image formed by the lens is in comet (or) egg shape such a defect is called coma.




- ⊗ Consider the lens is divided into different circular zones.
- ⊗ The light rays coming from the object 'A' passing through the zone (1, 1) will form the circular image at $I_1$.
- ⊗ Similarly the light rays passing through the zones (2, 2) and (3, 3) will form the circular images at $I_2$ and $I_3$ respectively.
- ⊗ As all the zones are in circular shape the image shapes are also circular.
- ⊗ If the radius of the circular zone increases, the radius of the circular shaped image also increases.
- ⊗ All these circular images are in different sizes and these images superimpose on one another at different positions. So, the final image formed is like a comet or egg.
- ⊗ Now let us see how the circular zone on the lens will form the circular image.
- ⊗ The light rays passing through the points (a, a) of the circular zone of the lens, will form the image at $a^1$. Similarly light rays through (b, b) and (c, c) form the images at $b^1$ and $c^1$ respectively. So ultimately the final image is like a circle.



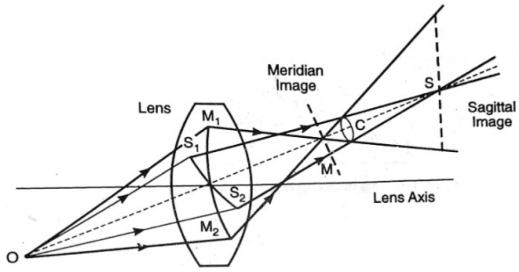

**Methods of minimization of coma**

1. **By using stops** :- The light rays coming from the object through the circular zones of higher radius are eliminated by using stops. So, the image is formed by the light coming from the circular zones of lower radius and the coma is minimized.
2. **By using crossed lens** :- If the refractive index of the material of the lens μ = 1.5, Then the shape factor will become $\beta = \frac{R_1}{R_2} = -\frac{1}{9}$ .
   Here $R_1$, $R_2$ are the radii of curvature of the two surfaces of the lens.
   Because of this, the shape of image the point size object is also in point size.

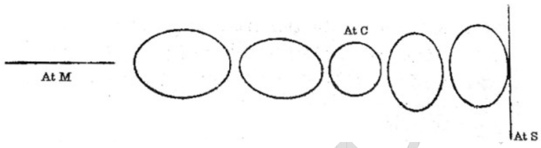
## Astigmatism

If the monochromatic object of point Size is placed far away from the axis of the lens, the image formed is not in point size and it is in different shapes along the axis, this type of defect is called astigmatism.

- Here are two planes.
  1. Meridional plane ($M_1M_2O$)., this plane contains the principal axis of the lens and the point object.
  2. Sagittal plane ($S_1S_2O$) , this plane is perpendicular to the meridional plane and also contains the point object.



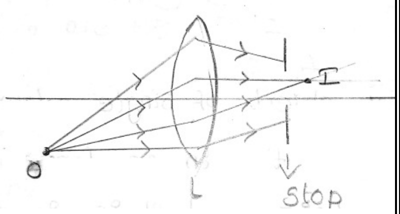

- The light rays coming from the point object "O" passing through meridional plane will form the image at near point M from the lens. Similarly the rays from the object "O" passinng through the sagittal plane will form the image at a far Point "S" from the lens.




- If a screen is moved from M to S the images of point object are as shown in the above figure. The shape of the image at point C is circle and this is called the "circle of least confusion". Here no image is in point size.

**<u>Methods of minimization of astigmatism</u>**

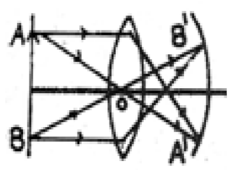
1. **<u>By using stops</u>** :- In order to eliminate the astigmatism, the stops are placed at a proper positions near the lens so that they can eliminate the rays with larger inclination.




2. The astigmatism is positive to convex lens and it is negative to concave lens. So, a convex lens and a concave lens of proper focal lengths are placed in contact as a doublet. By using this doublet astigmatism can be minimized. This lens combination is called astigmat. This is mostly used in cameras.
3. Astigmatism can be reduced by designing the lens in such a way that its curvatures along horizontal and vertical planes are of different values. This type of lens is called Toric lens.

## <u>Curvature of field</u> :-

The image of a plane object formed by the lens is in curved surface, the defect is called curvature of field.




In the above figure the object AB is in a plane but the image $A^1B^1$ formed by the lens is on a curved surface. The reason is that the light rays falling near to the axis ( paraxial rays) will form the image at a far distance. But the rays falling away from the axis ( marginal rays) will form the image at a near point from the lens. So, the image is formed in curved surface.

## <u>Minimization of curvature of a field</u> :-

- ☞ The curvature of field can be minimized by using thin lenses.
- ☞ The curvature of field can also be minimized by using stops.
- ☞ The curvature of field can be minimized by using coaxial lens system.

The radius of the curvature of field of coaxial lens system can be given by the following formula $\frac{1}{R} = \sum \frac{1}{f_n\ \mu_n}$ (or) $\frac{1}{R} = \frac{1}{f_1\mu_1} + \frac{1}{f_2\mu_2} + \ldots\ldots$

R = Radius of the curvature of field

f = Focal length of the lens

μ = Refractive index of the material of the lens

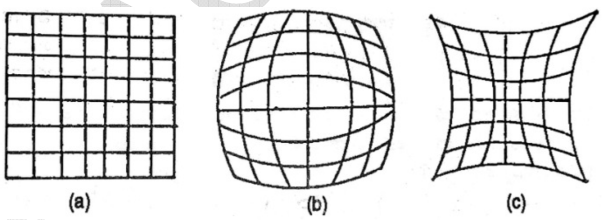
The curvature of field will be zero if the R value is infinite.

$$\frac{1}{\infty} = \frac{f_2\mu_2 + f_1\mu_1}{f_1\mu_1 f_2\mu_2} \quad \text{(or)} \quad f_2\mu_2 + f_1\mu_1 = 0$$

- The above equation is called petz wal's condition for minimization of curvature of field.
- Here $\mu_1, \mu_2$ are positive. In order to satisfy the above condition either $f_1$ or $f_2$ must be negative.
- That means one lens should be a convex lens and the other lens should be a concave lens.
- The distance between the two lenses is not a matter.

## **Distortion** :-

The image formed by the lens of a square object is not in the square shape, then the defect is called distortion.




The distortion is of two types.

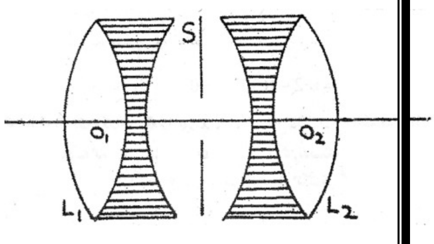
1) Barrel shaped distortion
2) Pin cushion shaped distortion

Barrel shaped distortion :- In this case the magnification of the lens near the principal axis is more than that at the edges of the lens. So, the image is barrel shaped (Fig. b) even though the object is in square shape (Fig. a).

Pincushion shaped distortion :- In this case the magnification of the lens near the principal axis is less than that at the edges of the lens. So, the image is pincushion shaped (Fig. c).

**Elimination of distortion** :-

1. The distortion can be minimized by using thin lenses.
2. To eliminate distortion a stop is placed in between two symmetrical lenses.




The lens system is shown in the above figure and is called arthroscopic doublet. The left doublet will create pincushion distortion where as the right doublet will create the barrel shaped distortion. These two distortions cancelled each other and the distortion is minimized.

# Chapter - II
# Interference -1

## (Wave-front division)

**Principle of superposition**:- If a particle of the medium is acted upon by two or more waves simultaneously, its resultant displacement is the algebraic sum of the displacements of the same particle due to individual waves in absence of the others.

Let $Y_1$ be the displacement taken by a particle due to the action of one wave and $Y_2$ be the displacement taken by the same particle due to the action of the other wave in the same direction, in absence of 1st wave, then the resultant displacement is $R = Y_1 + Y_2$

If the displacements are in opposite direction then the resultant displacement is $R = Y_1 - Y_2$

Combining the above two equations $\boxed{R = Y_1 \pm Y_2}$

## Interference of light

**Definition** :- The modification in the distribution of intensity of light in the region of superposition of two or more waves is called Interference of light.

Interference is of two types 1) Constructive interference and 2) Destructive interference

1) Constructive interference :- If the resultant amplitude is the sum of the amplitudes due to two waves, that interference is called constructive interference.

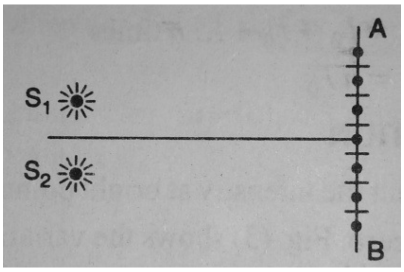
2) Destructive interference :- If the resultant amplitude is equal to the difference of the two amplitudes of the Waves, that interference is called destructive interference.

## Coherent sources and coherence

Coherent sources :- Two sources which produce the waves maintained at zero phase difference or at any constant phase difference between them are called coherent sources.

Coherence :- The phenomena of maintaining the two waves from two sources with zero phase difference or constant phase difference is called the coherence.

Explanation :- Let $S_1$ and $S_2$ be the two sources of light and AB is the screen in the **figure**.




- If the two waves starting from $S_1$ and $S_2$ are in the same phase i.e. the phase difference between them is zero, then all the dots on the screen are bright where as the dashes are dark.
- If the two waves starting from $S_1$ and $S_2$ are having $\pi$ phase difference, then all the dashes on the screen are bright where as the dots are dark.
- If the two Waves starting from $S_1$ and $S_2$ have a constant phase difference then also a stationary interference pattern is observed but the positions of bright and dark intensity points may change.
- If the phase difference between the two Waves coming from $S_1$ and $S_2$ changes randomly then stationery interference fringe pattern is not observed i.e. the intensities of bright and dark fringes change randomly and a uniform intensity is observed on the screen.
- So to get Stationary interference pattern the phase difference between the two sources must be constant, that means the two sources must be coherent sources.

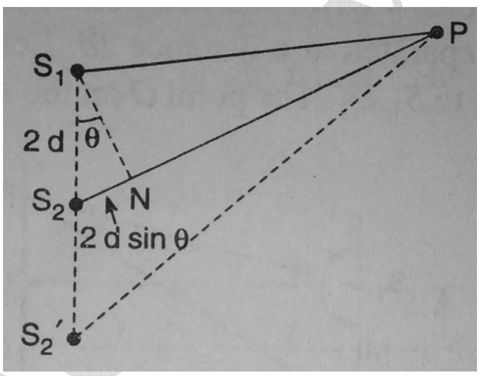
## Spatial coherence and temporal coherence

There are two types of coherences 1) Spatial coherence 2) Temporal coherence

**Spatial coherence**:-The method to obtain constant phase difference between two waves at the starting point from the sources is called spatial coherence.

Explanation :-

- The source of light emits wave train when its atom falls from excited state to ground state. The time taken for this is $10^{-9}$ sec and is called emission time.
- The path travelled by the wave train during this time is ($10^{-9}$xc) and it is equal to the length of the wave train ( $l = 10^{-9}xc$ ). Here c is the velocity of light in air.
- If the time difference between the two Wave trains coming from two sources $S_1$ and $S_2$ to the point of superposition (P) is less than $10^{-9}$ sec (or) the path difference between these two wave trains ($S_2P - S_1P = 2d \sin\theta$) is less than the length of the wave train, then only superposition is possible and interference takes place.




- If the time difference or the path difference between the wave trains is more than the above case, then the wave train from $S_1$ disappears at P before the wave train from $S_2$ reaches the point P. In this case superposition or interference is not possible even though their frequencies are equal.
- So, if the source $S_2$ is dragged away from $S_1$, upto $S^1_2$ then the path difference increases then the fringes loose their contrast and the sources are lack of spatial coherence.
- If, the width of the slit increases then also lack of spatial coherence takes place and the interference pattern looses contrast or it disappears.
- If the two sources are independent then also the sources loose their spatial coherence.

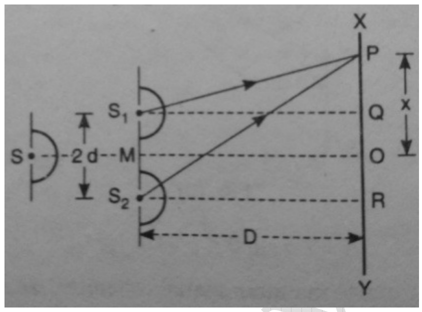
**Temporal coherence** :- The method to develop constant phase difference between two waves during their journey towards the point of superposition is called temporal coherence.

- The path difference between the two wave trains coming from two sources $S_1$ and $S_2$ can be increased or decreased by extending one of the paths.
- This is done by introducing a transparent sheet in one of the paths. This is called temporal coherence.
- If the path difference exceeds the length of the wave train then lack of temporal coherence occurs and the fringe pattern looses its contrast.

So to get stationery interference pattern with sufficient contrast, spatial coherence and temporal coherence are necessary.

## Theory of Interference fringes

- ✪ Consider a source S of a monochromatic light. Place two slits $S_1$ and$S_2$ at equidistance from S. The distance between the two slits is 2d.
- ✪ XY is the screen placed at distance D from the slits. The point O on the screen is at equidistance from $S_1$ and $S_2$.
- ✪ Bright fringe is formed at O because the path difference between the two waves coming from $S_1$ and $S_2$ to O is zero.
- ✪ P is the point where the intensity is to be calculated and it is at a distance '$x$' from point O.




The path difference (Δ) between the two waves coming from $S_1$ and $S_2$ to P is ($S_2P - S_1P$).

From the $\Delta^{le}$ $S_1QP$ $\quad (S_1P)^2 = (S_1Q)^2 + (QP)^2$

(or) $\quad (S_1P)^2 = D^2 + (x-d)^2 \quad [\because QP = (x-d)]$

Similarly from the $\Delta^{le}$ $S_2RP$ $\quad (S_2P)^2 = (S_2R)^2 + (RP)^2$

(or) $\quad (S_2P)^2 = D^2 + (x+d)^2 \quad [\because RP = (x+d)]$

Then from the above two equations

$$(S_2P)^2 - (S_1P)^2 = [\,D^2 + (x+d)^2\,] - [D^2 + (x-d)^2\,]$$

$$(S_2P)^2 - (S_1P)^2 = [(x+d)^2\,] - [(x-d)^2 \;= 4xd]$$

(or) $\quad (S_2P + S_1P)\,(S_2P - S_1P) = 4xd$

But $\quad (S_2P + S_1P) = 2D$

$\therefore \; 2D(S_2P - S_1P) = 4xd$

(or) Path difference $\quad \Delta = (S_2P - S_1P) = \frac{4xd}{2D} = \frac{2xd}{D} \longrightarrow$ (1)

<u>For bright fringes</u> :- To form bright band at P the path difference should be equal to nλ.

From eqn. (1) $\quad \Delta = \frac{2xd}{D} = n\lambda \quad$ (or) $\quad x = \frac{n\lambda D}{2d} \longrightarrow$ (2)

Here n = 0, 1, 2, …….

λ = wave length of the light used.

Distance of 1st bright band from point O $\quad x_1 = \frac{\lambda D}{2d} \quad$ n = 1

Distance of 2nd bright band from point O $\quad x_2 = \frac{2\lambda D}{2d} \quad$ n = 2

……………..

Similarly, distance of nth bright band from point O $\quad x_n = \frac{n\lambda D}{2d} \quad$ n = n

Distance between any two successive bright bands or band width $\beta = x_2 - x_1$

Substituting the above values

$$\beta = \frac{2\lambda D}{2d} - \frac{\lambda D}{2d} = \frac{\lambda D}{2d}$$ (or) Band width $\boxed{\beta = \frac{\lambda D}{2d}} \longrightarrow$ (3)

<u>For dark fringes</u> :- To form dark band at P the path difference should be equal to (2n+1)λ/2.

From eqn. (1) $\quad \Delta = \frac{2xd}{D} = (2n+1)\frac{\lambda}{2} \quad$ (or) $\quad x = (2n+1)\frac{\lambda D}{4d} \longrightarrow$ (4)

Here n = 0, 1, 2, …….

Distance of 1st dark band from point O $x_0 = \frac{\lambda D}{4d}$ n = 0

Distance of 2nd dark band from point O $x_1 = \frac{3\lambda D}{4d}$ n = 1

……………..

Similarly, distance of nth dark band from point O $x_n = (2n+1)\frac{\lambda D}{4d}$ n = n

Distance between any two successive dark bands or band width $\beta = x_1 - x_0$

Substituting the above values

$\beta = \frac{3\lambda D}{4d} - \frac{\lambda D}{4d} = \frac{\lambda D}{2d}$ (or) Band width $\boxed{\beta = \frac{\lambda D}{2d}}$ ⟶ (5)

From eqns. (3) and (5) width of bright band = width of dark band

## Conditions for interference of light

The conditions are of three types to get good interference pattern.

1) Conditions for sustained interference 2) Conditions for good observation 3) Conditions for good contrast

### 1) Conditions for sustained interference

I. The two sources must be coherent i.e. the phase difference (δ) between them must be constant. The resultant intensity of the interference fringe or band is $\boxed{I = (a_1)^2 + (a_2)^2 + 2\,a_1 a_2 \cos\delta}$

Here $a_1$ & $a_2$ are the amplitudes of the two individual light waves.

As per the above equation if δ is constant then the bright and dark bands are formed at fixed positions. If δ is not constant, then the intensities of the bright and dark bands changes continuously and finally it gives uniform intensity on the screen and no interference pattern is observed.

II. The two sources must emit continuous waves if the waves are discontinuous then superposition of two waves is not possible so sustained interference pattern is not possible.

### 2) Conditions for good obeservation

For good observation of the interference pattern the fringe width must be large. Then only we can observe the bands clearly.

The fringe width or band width is given by $\beta = \frac{\lambda D}{2d}$

Here λ = Wavelength of the light
D = Distance between the sources and the screen
2d = Distance between the two coherent sources

I. The distance between the sources and the screen should be large then β is large and the fringe pattern can clearly be seen.

II. The distance between the two coherent sources should be small then β is large. Then the bright and dark bands are distinguished.

III. The back-ground should be dark. Other wise the external light superimposed on the fringe pattern. So, the intensity difference between the dark and bright fringes is poor and they can not be distinguished.

### 3) Conditions for good contrast

If the edges of the fringes in the interference pattern are sharp then the fringe pattern is said to be with good contrast.

I. The amplitudes of the interfering waves should be equal.

Let $a_1$ & $a_2$ be the amplitudes of the two individual light waves, then the intensities of bright and dark bands are $(a_1 + a_2)^2$ and $(a_1 - a_2)^2$. If $a_1 = a_2$, then the dark band is perfectly black (intensity is zero) and the contrast is good.

II. The sources must be narrow. If they are wide, each slit acts as so many sources and different interference patterns formed & superimposed. Then the contrast is poor.

III. Monochromatic sources should be used. If composite light sources are used, the each colour will form its own fringe pattern. They overlap on one another and the contrast is poor.

## Classification of interference

The methods of formation of interference pattern are classified into two.

1) Division of wave front 2) Division of amplitude

1) Division of wave front:- Here the incident wave front is divided into two parts by the method of reflection, refraction diffraction. These two parts of the same wave front travel unequal distances and reunite to produce interference pattern.
Ex :- Fresnel biprism, Lloyd's mirror etc.

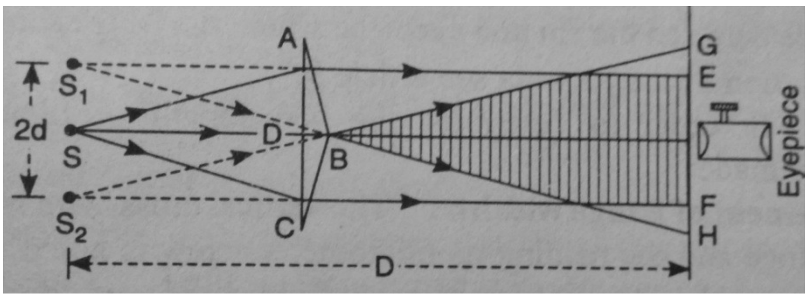
2) Division of amplitude :- Here the amplitude of the incoming light beam is divided into two parts. These two parts travel unequal distances and reunite to produce interference pattern.
Ex:- Newton's rings, Michelson's interferometer.

## Fresnel's biprism

- This prism is used to produce two coherent sources separated by some distance by using a single given slit.
- One of the angles of the biprism is obtuse and its value is $179^0$, the other two angles are $30^l$ each. This biprism is prepared by proper grinding and polishing an optically plane glass plate.

## Experimental arrangement

- This experiment is done on an optical bench. The optical bench carries three vertical stands. Their heights can be varied and they can be moved along the bench and perpendicular to the bench.
- An adjustable slit, biprism and micrometer eyepiece are placed on $1^{st}$ , $2^{nd}$ & $3^{rd}$ stands respectively. The slit and the biprism can be rotated in their own planes by using tangential screws.
- A monochromatic source of light is placed before the slit S, the light from the slit falls on the biprism, the edge B of the biprism divides the incident wave-front into two parts.
- One part appears to diverge from $S_1$ and the other part appears to diverge from $S_2$. Here $S_1$ and $S_2$ acts as two virtual coherent sources.



✪ The interference pattern is formed in the region of superposition.

**Adjustments**:-
1. Optical bench is made horizontal with the help of spirit level.
2. Slit, biprism and eyepiece are adjusted to the same height.
3. Eye-piece is adjusted so that the cross-wires are clearly seen.
4. The slit is made narrow and vertical.
5. The biprism is rotated with the tangential screw such that the fringes are clearly seen.
6. The biprism position and eyepiece position are adjusted in perpendicular to the optical bench such that the lateral shift is eliminated.

**Theory** :- The wavelength of monochromatic light is given by the formula

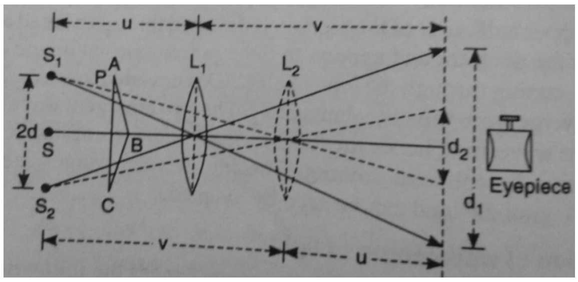
$$\boxed{\lambda = \frac{\beta.2d}{D}} \longrightarrow (1)$$

β = Fringe width.
2d = Distance between two virtual sources.
D = Distance between the slit and the eyepiece.

The above three quantities are measured in 3 individual steps.

Measurement of D :- The distance between the slit and the eyepiece (D) is measured with a metre scale or by taking the readings on the optical bench it self.

Measurement of β :- The vertical cross wire of the eyepiece is adjusted to coincide the left edge of any bright band by taking it as $0^{th}$ fringe. Now the eyepiece is moved perpendicular to the bench such that it coincides with the left edges of $5^{th}$,$10^{th}$, $15^{th}$… bright bands. Note the readings as $R_0$, $R_5$, $R_{10}$, $R_{15}$ … . The differences ($R_5$-$R_0$), ($R_{10}$-$R_5$), ($R_{15}$-$R_{10}$) are equal to 5 β. From these values the average 5 β and β are calculated.




Measurement of 2d :- In this case a convex lens is placed on another vertical stand between the biprism and eye-piece.

The focal length of the lens should be less than ¼ of the distance between slit and eye-piece.

If lens is slowly moved from the biprism towards the eyepiece, clear magnified and diminished images of coherent sources ($S_1$, $S_2$) are formed when the lens is at positions $L_1$ and $L_2$ respectively.

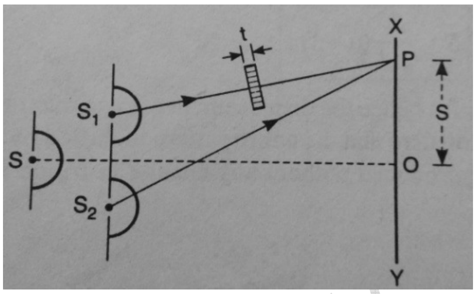
The distances between the two pairs of images measured as $d_1$ and $d_2$ respectively. Here the object distance (u)and image distances (v) are reversed when the lens position changes from $L_1$ to $L_2$ .

$$\therefore Magnification \quad \frac{d_1}{2d} = \frac{v}{u} \quad and \quad \frac{d_2}{2d} = \frac{u}{v}$$

Multiplying the two equations $4d^2 = d_1 d_2$ (or ) $\boxed{2d = \sqrt{d_1 d_2}}$

Substituting the values of D, β and 2d in equation (1) the wave length of the light (λ) can calculated.

## Determination of the thickness of a transparent sheet




- The biprism experiment can be used to determine the thickness of a thin transparent sheet like glass, Mica etc. Here $S_1$ and $S_2$are the two coherent sources. Point O is the central bright before introducing the sheet.
- After introducing the sheet of thickness t and refractive index μ, in $S_1O$ path, the central bright shifts from O to P through a distance S but the fringe width β does not change.
- The refractive index of the sheet $\mu = \frac{c_o}{c}$ (or ) $c = \frac{c_o}{\mu}$

Here $c_o$ & c = Velocity of light in air and sheet.

The path from S1 to P = $(S_1P - t)$ in air + (t) in material

The path in air $\Delta_1 = (S_1P - t) + \mu t = S_1P + (\mu - 1)t$

The path in air from $S_2$ to P $\Delta_2 = S_2P$

The path difference between the two rays $\Delta = \Delta_2 - \Delta_1 = [S_2P] - [S_1P + (\mu - 1)t]$

$$\Delta = [S_2P - S_1P] - (\mu - 1)t$$

But we know that $S_2P - S_1P = \frac{2xd}{D}$

. 2d = Distance between two sources.

D = Distance between the sources and the screen.

$$\Delta = \frac{2xd}{D} - (\mu - 1)t$$

For n[th] maxima in presence of the sheet $\Delta = \frac{2x_nd}{D} - (\mu - 1)t = n\lambda$

(or) $x_n = [n\lambda + (\mu - 1)t]\frac{D}{2d}$

For n[th] maxima in absence of the sheet (t = 0) $\therefore x'_n = [n\lambda]\frac{D}{2d}$

Displacement of the n[th] bright fringe $S = x_n - x'_n = [n\lambda + (\mu - 1)t]\frac{D}{2d} - [n\lambda]\frac{D}{2d}$

$$\boxed{S = \frac{D}{2d}(\mu - 1)t}$$

From the above equation the thickness of the transparent sheet can be calculated.

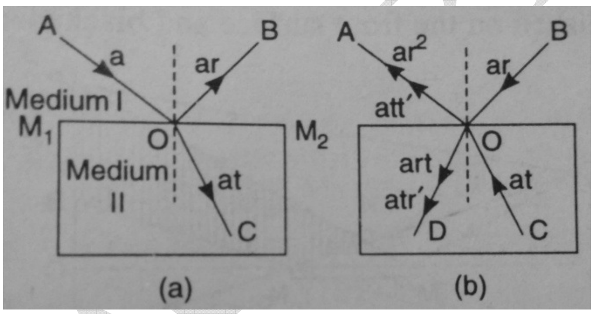
**Note** :- In monochromatic fringe pattern all the bright bands are in same colour and they can not be distinguished. The displacement is not known after introducing the . transparent sheet. But in white source fringe pattern, the central bright is white and all other bright bands are coloured. So, here the white source is taken instead of monochromatic source of light and the displacement of central bright fringe is measured.

## Change of phase on reflection

When a light ray reflects from the surface of a denser medium a phase change of π (or) a path difference of λ/2 takes place but if the reflection takes place at the surface rarer medium no such change takes place.

Explanation :-

- ✓ Let $M_1 M_2$ be the surface of separation of rarer medium –I and denser medium-II.
- ✓ Let r and $r^1$ be the reflection coefficients (fractions of the amplitudes reflected) in the medium-I and II respectively.
- ✓ Similarly t and $t^1$ are the transmission coefficients ( fractions of the amplitudes transmitted) from medium-I to II and from medium-II to I respectively.




- ✪ Consider the light wave 'AO' of amplitude 'a', incident at a point 'O'. A part of the ray reflected as 'OB' with amplitude 'ar' and a part is transmitted as you 'OC' with amplitude 'at' (Fig-a).
- ✪ If the 'OB' is reversed back then a part of the ray with amplitude '$ar^2$' is reflected in the path 'AO' and a part of the ray is transmitted as 'OD' with amplitude 'art'(Fig-b).
- ✪ Similarly, if the 'OC' is reversed back then a part of the ray with amplitude '$atr^1$' is reflected in the path 'OD' and a part of the ray is transmitted as 'OA' with amplitude '$att^1$'(Fig-b).

| | | |
|---|---|---|
| The total amplitude along OD | $art + atr^1 = 0 \longrightarrow$ (1) | ∵ No ray is observed along OD |
| The total amplitude along OA | $ar^2 + att^1 = a$ | ∵ The total energy is reversed to the path OA |

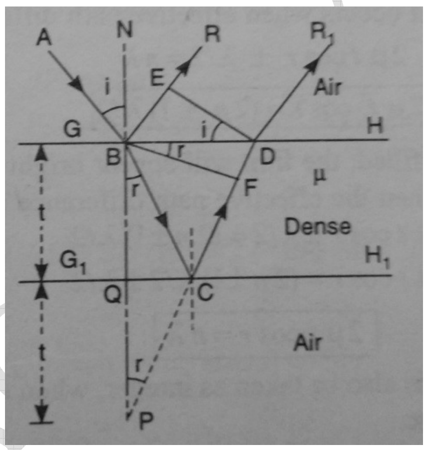
From the eqn. (1) $\boxed{r = -r'} \longrightarrow$ (2)

- ☞ The negative sign in equation (2) indicates that the phase change of π occur due to the reflection either at the denser surface or at the rarer surface.
- ☞ But in Lloyd's mirror experiment the interference takes place between the direct ray and the ray reflected at the denser surface. There the phase reversal occurred.
- ☞ So, from the above discussion, it is clear that the phase reversal takes place only when the ray is reflected at the denser medium but not at the rarer medium.

# Chapter - III
# Interference -2
(Amplitude division)

## Oblique incidence of plane wave on thin film - cosine law (Reflected light)

- Consider a thin transparent sheet of uniform thickness 't' and refractive index 'μ'. Its upper and lower surfaces are GH and $G_1 H_1$. A ray AB incident on the upper surface of the film with angle of incidence 'i'.
- A part of the ray is reflected as BR and a part of the ray is refracted with angle 'r' into the second medium as BC.
- At the second surface the BC is reflected as CD and the finally the ray emerges out into air as $DR_1$. BR and $DR_1$ are parallel and interfere.
- Draw a normal BF onto CD and another normal DE on to BR. The Ray CD is extended back and it cuts the normal NQ at the point P.
- From the figure the angles are as follows
  $\angle ABN = i$, angle of incidence
  $\angle QBC = r$, angle of refraction
  $\angle BDE = i$, $\angle DBF = r$, $\angle QPC = r$




The path difference between the two reflected rays i.e. BR and $DR_1$.

$$\Delta' = \text{Path } (BC + CD) \text{ in medium} - \text{Path } (BE) \text{ in air}$$

$$\therefore \Delta' = \mu (BC + CD) - (BE) \longrightarrow (1)$$

From the two $\Delta^{les}$ EDB & FBD and

as per Snell' law the refractive index of the film $\mu = \frac{\sin i}{\sin r} = \frac{BE/BD}{FD/BD} = \frac{BE}{FD}$

$$\therefore BE = \mu FD \longrightarrow (2)$$

Substituting eqn. (2) in (1) $\Delta' = \mu (BC + CD) - \mu (FD)$

$$\Delta' = \mu [BC + (CF + FD] - \mu (FD)$$

$$\Delta' = \mu (BC + CF)$$

$$\Delta' = \mu (PF) \quad [\because BC = CP] \longrightarrow (3)$$

From the $\Delta^{le}$ BFP $\cos r = PF/_{BP}$

(or) PF = BP cos r (or) $PF = 2t.\cos r$ $[\because BP = 2t] \longrightarrow (4)$

Substituting eqn. (4) in (3) $\Delta' = 2\mu t \cos r$

But the ray BR reflected at denser surface (GH) takes additional phase difference of $\pi$ or path difference of $\lambda/2$.

So, the actual path difference $\Delta = \Delta' - \lambda/_2$ or $\boxed{\Delta = 2\mu t \cos r - \lambda/_2}$

**Condition for bright band**

$\Delta = n\lambda$ (or) $2\mu t \cos r - \lambda/_2 = n\lambda$ ( n = 0,1,2,3… = No. of the fringe )

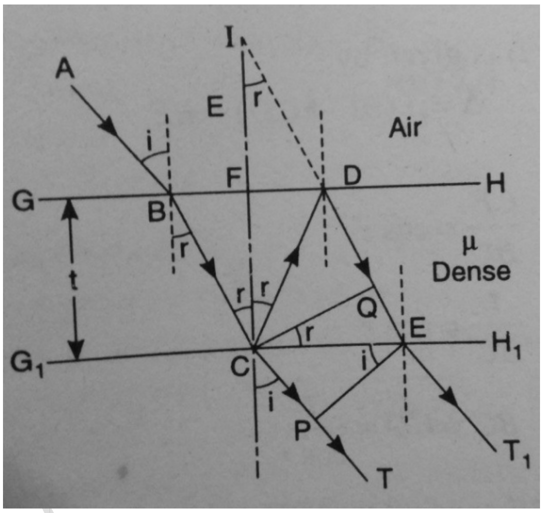
$$\boxed{2\mu t \cos r = (2n + 1)\frac{\lambda}{2}}$$

**Condition for dark band**

$\Delta = (2n+1)\frac{\lambda}{2}$ (or) $2\mu t\cos r - \lambda/2 = (2n+1)\frac{\lambda}{2}$ ( n = 0,1,2,3… = No. of the fringe )

$$\boxed{2\mu t\cos r = (n+1)\lambda} \quad \text{(or)} \quad \boxed{2\mu t\cos r = n\lambda}$$

**Oblique incidence of plane wave on thin film - cosine law (Transmitted light)**




- Consider a thin transparent sheet of uniform thickness 't' and refractive index 'μ'. Its upper and lower surfaces are GH and $G_1 H_1$. A ray AB incident on the upper surface of the film with angle of incidence 'i'.
- The ray is refracted with angle 'r' into the second medium as BC.
- At the second surface, a part of the ray BC is emerged out as CT with angle 'i' and a part of the ray is reflected as CD with angle 'r' and this ray once again reflected back as DE with same angle 'r'. This ray finally emerges out into air as $ET_1$. CT and $ET_1$ are parallel and interfere.
- Draw a normal EP onto CT and another normal CQ on to DE. The ray ED is extended back and it cuts the normal CF at the point I.
- From the figure the angles are as follows
  ∠CEP = i, angle of incidence
  ∠FCD = r, angle of refraction
  ∠QCE = r, ∠FID = r

The path difference between the two rays i.e. CT and $ET_1$.

Δ = Path (CD + DE) in medium – Path (CP) in air

∴ Δ= μ (CD + DE) – (CP) ⟶ (1)

From the two $\Delta^{les}$ CPE & CQE and

as per Snell' law the refractive index of the film $\mu = \frac{\sin i}{\sin r} = \frac{CP/CE}{QE/CE} = \frac{CP}{QE}$

$\therefore\ CP = \mu\, QE$ ⟶ (2)

Substituting eqn. (2) in (1) $\Delta = \mu\,(CD + DE) - \mu\,(QE)$

$\Delta = \mu\,[CD + (DQ + QE] - \mu\,(QE)$

$\Delta = \mu\,(CD + DQ)$

$\Delta = \mu\,(QI)$ [ ∵ CD = ID ] ⟶ (3)

From the $\Delta^{le}$ CQI $\cos r = QI/CI$

(or) QI = CI cos r (or) $QI = 2t.\cos r$ [ ∵ CI = 2t ] ⟶ (4)

Substituting eqn. (4) in (3) $\boxed{\Delta = 2\mu t\cos r}$

**Note** :- For the reflections with in the denser medium, the two surfaces acts as rarer surfaces. So, no phase reversal takes place in these reflections

**Condition for bright band**

$\Delta = n\lambda$ (or) $\boxed{2\mu t \cos r = n\lambda}$ ( n = 0,1,2,3… = No. of the fringe )

**Condition for dark band**

$\Delta = (2n+1)\frac{\lambda}{2}$ (or) $\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}}$ ( n = 0,1,2,3… = No. of the fringe )

Complementary character of reflected and transmitted interference pattern

The conditions for constructive interference and destructive interference are reversed in reflected and transmitted interference patterns.

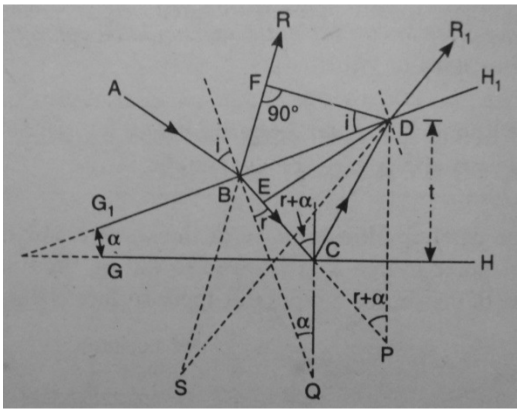
## Colours of thin films

- When white light incident on a thin parallel film, different colours are observed.
- The incident white light splits into its constituent colours when refracted.
- These light rays reflected from upper and lower surfaces interfere each other because of this different colours are observed.
- Even though the thickness of the film 't' is constant for all colours, the angle of refraction 'r' and the refractive index 'μ' depends on the colour.
- If a particular colour satisfies the equation $\boxed{2\mu t \cos r = n\lambda}$ then that fringe appears in that colour and all other colours are absent (dark).
- If one point on the film is observed from different positions then the angle of refraction 'r' changes and the colour also changes. Similarly if different points on the film are observed from the same position then also for each point the angle of refraction 'r' changes and the colour also changes.
- When oil film is formed on water surface then the two surfaces of film will act as denser surfaces because the refractive index of oil is in between the refractive indices of air and water. So, at each surface the phase reversal takes place during reflection and they cancel each other.
- So, the equation for the bright band is $\boxed{2\mu t \cos r = n\lambda}$ . This is the same equation as that of interference of transmitted light even though the colours are formed due to reflection of light.

## Interference by a non-parallel film or wedge shaped film

Consider a film of refractive index μ. Its surfaces GH and $G_1H_1$ are non parallel and the angle between them is α..

The thickness of the film increases from one edge to the other and is called wedge shaped film. The thickness at the second edge is 't'



- ☞ A ray AB incident on the upper surface of the wedge film with angle of incidence 'i'.
- ☞ A part of the ray is reflected as BR and a part of the ray is refracted with angle 'r' into the second medium as BC.
- ☞ At the second surface the BC is reflected as CD and the finally the ray emerges out into air as $DR_1$. The rays BR and $DR_1$ appear to diverge from P and interfere.
- ☞ Draw a normal DF onto BR and another normal DE on to BC. The ray BC is extended and it cuts the normal DP at the point P.
- ☞ From the figure the angles are as follows

  $\angle BDF = i$, angle of incidence

  $\angle BDE = r$, angle of refraction

  $\angle CDP = \angle CPD = (r+\alpha)$ , CD = CP and DP = 2t

The path difference between the two reflected rays i.e. BR and $DR_1$.

$\Delta'$ = Path (BC + CD) in medium – Path (BF) in air

$$\therefore \ \Delta' = \mu\,(BC + CD) - (BF) \longrightarrow (1)$$

From the two $\Delta^{les}$ BFD & BED

As per Snell' law the refractive index of the film $\mu = \frac{\sin i}{\sin r} = \frac{BF/BD}{BE/BD} = \frac{BF}{BE}$

$$\therefore \ BF = \mu\, BE \longrightarrow (2)$$

Substituting eqn. (2) in (1)

$$\Delta' = \mu\,(BC + CD) - \mu\,(BE)$$

$$\Delta' = \mu\,[(BE + EC) + CD] - \mu\,(BE)$$

$$\Delta' = \mu\,(EC + CD)$$

$$\Delta' = \mu\,(EP) \quad [\ \because CD = CP\ ] \longrightarrow (3)$$

From the $\Delta^{le}$ DEP $\cos(r+\alpha) = {EP}/{DP}$

(or) $EP = DP\cos(r+\alpha)$ (or) $EP = 2t.\cos(r+\alpha)$ $[\ \because DP = 2t\ ] \longrightarrow (4)$

Substituting eqn. (4) in (3) $\Delta' = 2\mu t\cos(r+\alpha)$

But the ray reflected at denser surface ($G_1H_1$) takes additional phase difference of $\pi$ or path difference of $\lambda/2$.

So, the actual path difference $\Delta = \Delta' - \lambda/2$ or $\boxed{\Delta = 2\mu t\cos(r+\alpha) - \lambda/2}$

**<u>Condition for bright band</u>**

$\Delta = n\lambda$ (or) $2\mu t\cos(r+\alpha) - \lambda/2 = n\lambda$ ( n = 0,1,2,3… = No. of the fringe )

$$\boxed{2\mu t\cos(r+\alpha) = (2n+1)\frac{\lambda}{2}}$$

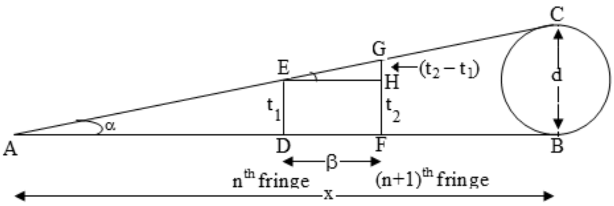
**<u>Condition for dark band</u>**

$\Delta = (2n+1)\frac{\lambda}{2}$ (or) $2\mu t\cos(r+\alpha) - \lambda/2 = (2n+1)\frac{\lambda}{2}$ (n = 0,1,2,3… = No. of the fringe )

$$\boxed{2\mu t\cos(r+\alpha) = (n+1)\lambda} \quad \text{(or)} \quad \boxed{2\mu t\cos(r+\alpha) = n\lambda}$$

**Determination of the angle of the wedge (or) diameter of the wire**

Two rectangular optically plane glass plates are taken. The wire whose diameter 'd' is to be determined is placed between (BC) the plates at one end. At the other end 'A' the two plates are in contact. The angle between them is 'α'. The wedge shaped air film is formed between AB and AC.




- When a light ray incident normally on the wedge, a part of the ray reflected from lower surface of upper glass plate and a part of the ray reflected from upper surface of the lower glass plate.
- These two parts interfere each other to form fringes. These fringes are straight and parallel to edge where the two glass plates are in contact.

Let $t_1$ and $t_2$ be the thicknesses of air film at D and F where $n^{th}$ and $(n+1)^{th}$ dark fringes are formed. Then DF = β.

A line EH parallel to DF is drawn. So, ∠GEH = α and GH = $(t_2 - t_1)$

The conditions for dark bands at D and F are

$2t_1 = n\lambda$ $\quad \because 2\mu t \cos r = n\lambda$ Here μ =1 for air & cos r =1 for normal incidence

Similarly $2t_2 = (n+1)\lambda$

Then $2t_2 - 2t_1 = (n+1)\lambda - n\lambda$ (or) $(t_2 - t_1) = \lambda/2 \longrightarrow (1)$

From the $\Delta^{le}$ EGH $\quad \tan\alpha = \frac{GH}{EH}$

$$\tan\alpha = \frac{(t_2 - t_1)}{\beta} \longrightarrow (2) \quad \because GH = (t_2 - t_1) \;\&\; DF = EH = \beta$$

From eqns. (1) and (2) $\boxed{\tan\alpha = \frac{\lambda}{2\beta}} \longrightarrow (3)$

From the eqn. (3) the angle of the wedge can be calculated.

But from the $\Delta^{le}$ABC,

AB = $X$ = Distance between the wire and the contact edge of the glass plates.

BC = d = diameter of the wire

$$\tan\alpha = \frac{BC}{AB} = \frac{d}{x} \longrightarrow (4)$$

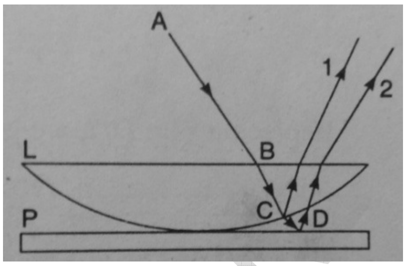
Comparing the eqns. (3) and (4)

$$\frac{d}{x} = \frac{\lambda}{2\beta} \quad \text{(or)} \quad \boxed{d = \frac{\lambda x}{2\beta}}$$

From this eqn. the diameter of the wire 'd' can be calculated.

# Newton's rings

- ✓ A plano-convex lens of large radius of curvature, is placed on a plane glass plate such that the convex surface touches glass plate. An air film is formed between the lens and glass plate. Its thickness increases as it goes from centre to edge.
- ✓ So, concentric circular fringes, alternate bright and dark fringes, are formed when monochromatic light incident normally on the lens.
- ✓ These fringes are called Newton's rings.




**Theory** :-

- ⊙ When a monochromatic light ray (AB) incident normally on the lens (For explanation purpose some inclination is shown) a part of the ray is reflected from the lower surface of the lens as ray-1 and a part of the ray is reflected from the upper surface of the glass plate as ray -2. These two rays – 1 & 2, interfere each other.
- ⊙ As ray -2 is reflected at denser surface it takes additional phase difference of $\pi$ or a path difference of $\lambda/2$ .

  Let 't' be the thickness of the air film at that point.

So, the path difference between the two rays $\Delta = 2\,\mu\, t \cos r + {}^{\lambda}/_{2}$

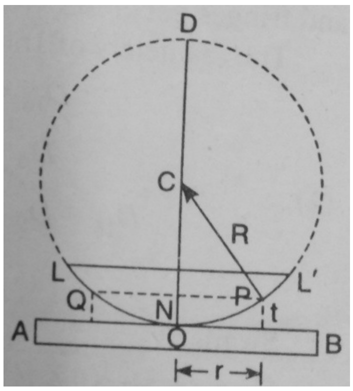
$\mu = 1$ for air and $r = 0$ (or) $\cos r = 1$ for normal incidence

Then, $\boxed{\Delta = 2\,t + {}^{\lambda}/_{2}}$ ⟶ (1)

At the centre, the lens is in contact with the glass plate, there $t = 0$ & $\Delta = {}^{\lambda}/_{2}$

This is the condition for dark fringe. So, the central spot (fringe) is dark for reflected light.

Diameter of the ring




- ✓ In the figure, AB is the glass plate & LOL[1] is the convex surface of the plano-convex lens. This is the part of the sphere of radius R. i.e. R is the radius of curvature of the lens.
- ✓ Let 't' be the thickness of the air film at P & Q which are at a distance 'r' from the centre 'O'. This 'r' is equal to the radius of the ring.

From eqn.(1) condition for bright band is

$2\,t + {}^{\lambda}/_{2} = n\lambda$ (or) $2\,t = (2n-1)\,{}^{\lambda}/_{2}$ ⟶ (2)

Condition for dark band is

$2\,t + {}^{\lambda}/_{2} = (2n+1)\,{}^{\lambda}/_{2}$ (or) $2\,t = n\lambda$ ⟶ (3) where n = 0, 1, 2, 3 …. etc.

From the property of the circle, QN x NP = ON x ND ⟶ (4)

But QN = NP = r = Radius of the ring

ON = t = Thickness of the air film

DN = (2R – t ) ∵ OC = CD = R = Radius of curvature of convex surface.

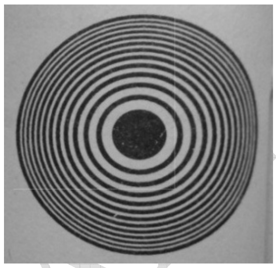
Substituting the above values in eqn. (4)

r x r = (2R - t) x t (or) $r^2 = 2Rt - t^2$

But 't' is small and $t^2$ value can be neglected.

∴ $r^2 = 2Rt$ (or) $\left(\frac{D}{2}\right)^2 = 2Rt$ ∴ D = Diameter of the ring

∴ $2t = \frac{D^2}{4R}$ ⟶ (5)



Newton's rings – reflected light

Condition for bright ring

Substituting eqn. (5) in (2)

$\frac{D^2}{4R} = (2n-1)\,\lambda/_2$ (or) $\boxed{D^2 = (2n-1)\,2\lambda R}$

$D^2 \propto (2n-1)$ (or) $D \propto \sqrt{(2n-1)}$

So, the diameter of the bright ring is directly proportional to odd natural number.

Condition for dark ring

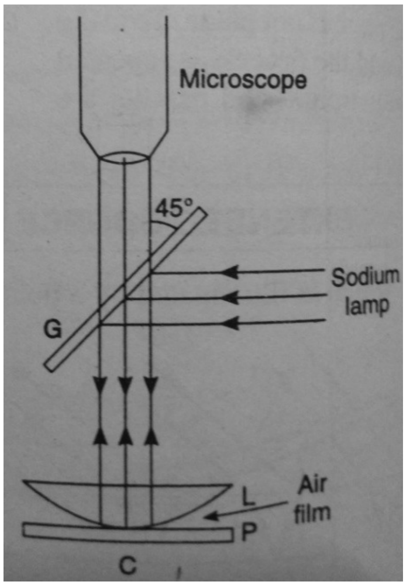
Substituting eqn. (5) in (3) $\frac{D^2}{4R} = n\lambda$ (or) $\boxed{D^2 = 4n\lambda R}$

$D^2 \propto n$ (or) $D \propto \sqrt{n}$

So, the diameter of the dark ring is directly proportional to natural number.

## Determination of the wave length of monochromatic light using Newton's rings

Experimental arrangement




- ✪ A Plano-convex lens of larger radius of curvature (R), is placed on a optically plane glass plate (P) such that the convex surface of the lens touches the glass plate.
- ✪ Below the glass plate (P) a black paper is placed so that it avoids the reflection at the bottom surface of the glass plate.
- ✪ Light from monochromatic source is made to fall on another glass plate (G) placed at an angle $45^0$ with the incident light. This glass plate reflects the light and this light incident normally on the lens.
- ✪ A part of the light is reflected at the curved surface of the lens and a part is transmitted into air film and reflected back from the upper surface of the glass plate (P).
- ✪ These two reflected lights interfere each other giving interference pattern in the form of circular rings. The fringe width decrease from the centre to the edge of the lens because the wedge angle (α) increases from centre to the edge.
- ✪ These rings are observed by a travelling microscope placed vertically above the inclined glass plate(G).

Theory :- The diameters of the $n^{th}$ and $m^{th}$ dark rings are given by

$$D_n^2 = 4n\lambda R$$

$$D_m^2 = 4m\lambda R \quad \text{Here} \quad \lambda = wave\ length\ of\ light$$

R = Radius of curvature of the convex surface of the lens.

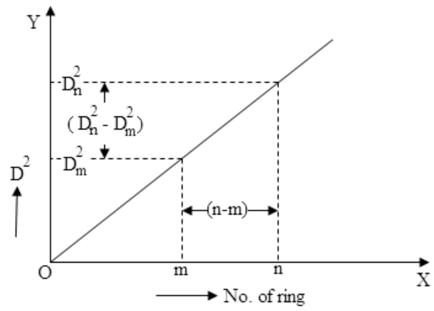
Then $D_n^2 - D_m^2 = 4(n-m)\lambda R$ (or) $\boxed{\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}}$ ⟶ (1)

From this equation the wave length of the monochromatic light (λ) can be determined.

Procedure :-

- The eyepiece of the microscope is focused to the centre of the fringe pattern. Now the microscope is moved to the extreme left of the fringe pattern.
- The vertical cross-wire is adjusted to coincide with the $n^{th}$ dark fringe and the microscope is moved to coincide with (n – 2) and (n-4) etc. up to $10^{th}$ dark fringe. The readings on microscope are taken as $R_n$, $R_{(n-2)}$, $R_{(n-4)}$, … $R_{10}$ etc.
- Now the microscope is moved to right side of the pattern and the vertical cross-wire is adjusted to coincide with $10^{th}$, … $(n-4)^{th}$, $(n-2)^{th}$ and $n^{th}$ dark fringes. The readings on microscope are taken as $R^1_{10}$… $R^1_{(n-4)}$, $R^1_{(n-2)}$ and $R^1_n$.
- The differences $(R_n \sim R^1_n)$, $(R_{(n-2)} \sim R^1_{(n-2)})$, $(R_{(n-4)} \sim R^1_{(n-4)})$, $(R_{10} \sim R^1_{10})$ give the diameters of $n^{th}$, $(n-2)^{th}$, $(n-4)^{th}$ … $10^{th}$ dark rings respectively.

Graph :- A graph is drawn by taking no. of the dark ring on X-axis and square of the diameter on Y-axis. The graph is a straight line passing through origin. The difference $(D_n^2 - D_m^2)$ is taken for the difference of (n-m) is taken.




The radius of curvature (R) of the convex surface of the Plano-convex lens is measured with spherometer by using the formula $\boxed{R = \frac{l^2}{6h} + \frac{h}{2}}$

Where

l = Average distance between the legs of spherometer.

h = Curved height.

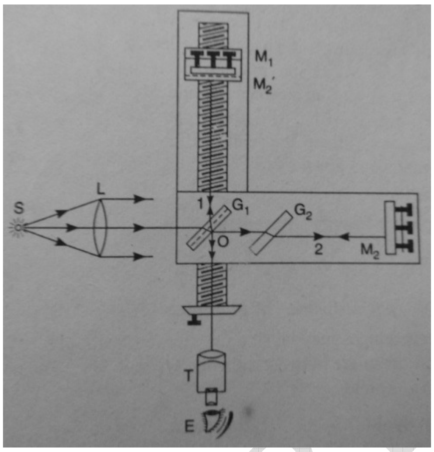
by substituting these values in eqn. (1) the wave length (λ) of the monochromatic light is determined.

## Michelson interferometer

**Construction** :-

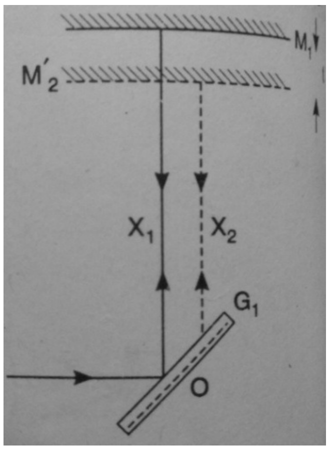
- This interferometer consists of a L shaped frame. On one arm a highly polished mirror $M_1$ is placed on a carriage and on the other arm another mirror $M_2$ is placed.
- These two Mirrors have 3 leveling screws each. The angle between the mirrors $M_1$ & $M_2$ can be changed by using the leveling screws. These two mirrors are made perfectly perpendicular to each other.
- The mirror $M_1$ can be moved along the arm on the carriage by using the micrometer screw.

- An optically plane glass plate $G_1$ semi silvered on its second surface, is placed at an angle $45^0$ with the incident beam of monochromatic light.
- This glass plate $G_1$ also makes $45^0$ angle with the mirrors $M_1$ and also with $M_2$.
- A telescope (T) is placed to receive the light rays from $G_1$ which were reflected from $M_1$ and $M_2$.




**Working** :-

- ✓ Light from a monochromatic source is made parallel and narrow. This narrow beam of light incident on glass plate $G_1$.
- ✓ At the semi silvered surface of $G_1$ the light ray is divided into two parts. One part is reflected as ray-1 and incident normally on $M_1$ and reflected back.
- ✓ The second part is transmitted as ray-2 and incident normally on $M_2$ and there it is reflected back.
- ✓ These two reflected rays from $M_1$ and $M_2$, meet at semi-silvered surface of $G_1$ with some path difference and interfere. This interference pattern is observed through the telescope(T).
- ✓ The path difference between the two rays can be changed by moving the mirror $M_1$.
- ✓ Here the ray-1 passes twice through $G_1$ but ray-2 does not pass even once. To compensate this path difference between the two rays, another glass plate $G_2$ of same dimensions and same material as that of $G_1$ and with out silver coating, is placed in the path of ray-2 parallel to $G_1$. So the glass-plate $G_2$ is called compensating glass plate.
- ✓ Introduction of $G_2$ is compulsory for white light but for monochromatic light it is not necessary.
- ✓ When observed through the telescope, the rays reflected from $M_2$ appears as it is reflected from $M_2^1$. $M_1$ and $M_2^1$ are parallel.




If $X_1$ and $X_2$ are the distances of $M_1$and $M_2^1$ from the glass plate $G_1$,

Then the path difference between the two rays is given by

$$\boxed{\Delta = 2\,(X_2 - X_1)}$$ for thick silver coating of $G_1$

And $$\boxed{\Delta = 2\,(X_2 - X_1) \pm \frac{\lambda}{2}}$$ for thin silver coating of $G_1$

**Determination of the wave length of the monochromatic light using Michelson's interferometer**

The mirrors $M_1$ and $M_2$ are adjusted by using the leveling screws such that circular fringes are seen through the telescope.
If 't' is the thickness of the air film between $M_1$ and $M_2^1$ then, condition for $n^{th}$ order central bright spot is

$$2\mu t \cos r + \lambda/2 = n\lambda$$

For air film $\mu = 1$ and for normal incidence $r = 0^0$ and $\cos r = \cos 0^0 = 1$

$$\therefore\ 2t + \lambda/2 = n\lambda$$

Note the position of $M_1$ as $X_1$ by using micrometer. Now move the mirror $M_1$ for a distance $\lambda/4$ away from $M_2^1$ then the path difference $\lambda/2$ increases and central spot becomes dark. For another $\lambda/4$ displacement of $M_1$ the path difference $\lambda/2$ increases and central spot becomes bright. Then one bright spot is said to be collapsed. It means that for each $\lambda/2$ displacement of $M_1$ one fringe is collapsed.
Move the mirror $M_1$ such that N central bright spots are collapsed and finally note the position of $M_1$ as $X_2$.

Then $N\frac{\lambda}{2} = (X_2 - X_1)$ (or) $\boxed{\lambda = \frac{2(X_2-X_1)}{N}}$

Repeat the experiment for no. of times and take the average value of $\lambda$.

# Chapter – IV

## Diffraction – 1 (Fraunhofer's diffraction)

Diffraction definition :- If the size of the obstacle placed in the path of light is small and is comparable to the size of the wave length of the light used, the light ray bends into the geometrical shadow region of the obstacle.

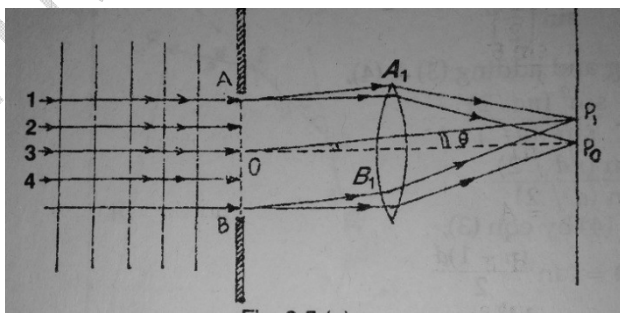
The bending of light in to the geometrical shadow region of the obstacle is called diffraction.

Diffractions are of two types. 1) Fraunhofer's diffraction 2) Fresnel's diffraction.

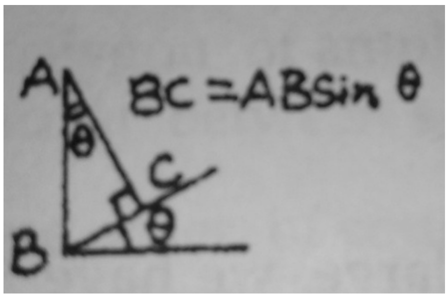
**Differences between Fraunhofer's and Fresnel's diffractions**

| S.No. | Fraunhofer's diffraction | Fresnel's diffraction |
|---|---|---|
| 1. | The source of light and the screen are placed at infinite distance from the obstacle. | The source of light and the screen are placed at finite distance from the obstacle. |
| 2. | In Fraunhofer's diffraction the wave front is plane wave front. | In Fresnel's diffraction the wave front is spherical or cylindrical wave front. |
| 3. | In this diffraction the light rays are parallel. | In this diffraction the light rays are non-parallel. |
| 4. | In this diffraction, lenses or the lens system is used. | In this diffraction, lenses are not used. |
| 5. | In this diffraction the mathematical analysis is simple. | In this diffraction the mathematical analysis is complicated. |

**Fraunhofer diffraction due to single slit**




- ✪ A parallel beam of light rays (1, 2, 3 & 4) are incidenting normally on a slit AB of width 'e'. These rays form plane wave front.
- ✪ If there is no diffraction at the edges of the slit, then the convex lens converges these rays to form the image at the point '$P_o$' on the screen. The screen is placed in the focal plane of the lens. The image formed at $P_o$ has maximum intensity.
- ✪ The diffracted rays $AA_1$, $BB_1$ at the slit travel with deviation 'θ' and form the image at $P_1$.
- ✪ In order to find out the path difference between $AA_1$ & $BB_1$ a perpendicular AC should be drawn from the point 'A' on to $BB_1$.




Slit width AB = e

From the $\Delta^{le}$ ABC $\quad Sin\,\theta = \frac{BC}{AB}$

$\therefore BC = (AB) Sin\,\theta$

(or) Path difference $BC = e\,Sin\,\theta$

(or) Phase difference $= \frac{2\pi}{\lambda}(e\,Sin\,\theta)$

If the slit is divided into 'n' equal parts, the phase difference in each part be 'd'. Then $d = \frac{1}{n}.\frac{2\pi}{\lambda}(e\,Sin\,\theta) \longrightarrow$ (1)

We know that

the resultant amplitude of 'n' simple harmonic motions $R = \frac{a\,Sin(\frac{nd}{2})}{Sin(\frac{d}{2})} \longrightarrow$ (2)

Substituting eqn. (1) in (2)

Then $R = \frac{a\,Sin(\frac{n\frac{1}{n}\cdot\frac{2\pi}{\lambda}(e\,Sin\,\theta))}{2})}{Sin(\frac{\frac{1}{n}\cdot\frac{2\pi}{\lambda}(e\,Sin\,\theta)}{2})} = \frac{a\,Sin[\frac{\pi}{\lambda}(e\,Sin\,\theta)]}{Sin[\frac{1}{n}\frac{\pi}{\lambda}(e\,Sin\,\theta)]} \longrightarrow$ (3)

Put $\frac{\pi}{\lambda}(e\,Sin\,\theta) = \alpha$

Then eqn. (3) becomes $R = \frac{a\,Sin\,\alpha}{Sin[\frac{\alpha}{n}]}$

As n value is large, then $\frac{\alpha}{n}$ is too small, then $Sin\left[\frac{\alpha}{n}\right] = \frac{\alpha}{n}$

Then $R = \frac{a\,Sin\,\alpha}{\frac{\alpha}{n}} = \frac{na\,Sin\,\alpha}{\alpha}$

Put $na = A$

$$\therefore R = \frac{A\,Sin\,\alpha}{\alpha}$$

When $n \to \infty$ and $\alpha \to 0$ then $A = na = finite$

Intensity of light $I = R^2 = \frac{A^2\,Sin^2\,\alpha}{\alpha^2} = \frac{I_o\,Sin^2\,\alpha}{\alpha^2}$ Here $A^2 = I_o$

But $Sin\,\alpha = \frac{\alpha}{1!} - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} \ldots\ldots$

$\therefore R = \frac{A}{\alpha}.Sin\,\alpha = \frac{A}{\alpha}\left[\frac{\alpha}{1!} - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} \ldots\ldots\right]$

$R = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} \ldots\ldots\right]$

Condition for maxima

In order to get maximum intensity, the value of 'R' should be maximum.

Then, $\alpha$ should be equal to zero. $\alpha = 0$

Then, R =A and $I = R^2 = A^2 = I_o$

$\alpha = 0$ means $\frac{\pi}{\lambda}(e\,Sin\,\theta) = 0$ $(or)$ $\theta = 0$

The rays with $\theta = 0$ means, the rays which are travelling with out deviation. These rays form the image at point $P_o$ and this has maximum intensity.

This maximum is called principal maximum (or) central maximum.

Condition for minima

$$I = \frac{A^2\,Sin^2\,\alpha}{\alpha^2}$$

To get minimum intensity $Sin\,\alpha = 0$ and $\alpha \neq 0$

Then $\alpha = \pm m\pi$ (or) $\frac{\pi}{\lambda}(e\,Sin\,\theta) = \pm m\pi$

$(e\,Sin\,\theta) = \pm m\lambda$ i.e. $\pm\lambda, \pm 2\lambda, \pm 3\lambda \ldots\ldots$

This is the condition for minimum intensity.

$\pm$ means the minima are formed on both sides of Principal maxima.

In between two minima there should be maxima and those maxima are called secondary maxima.

Condition for secondary maxima

$$I = \frac{A^2\, Sin^2\,\alpha}{\alpha^2}$$

$$I = Maximum \quad if \quad \frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left(\frac{A^2\, Sin^2\,\alpha}{\alpha^2}\right) = A^2 \frac{d}{d\alpha}\left(Sin^2\,\alpha\,.\,\alpha^{-2}\right) = 0$$

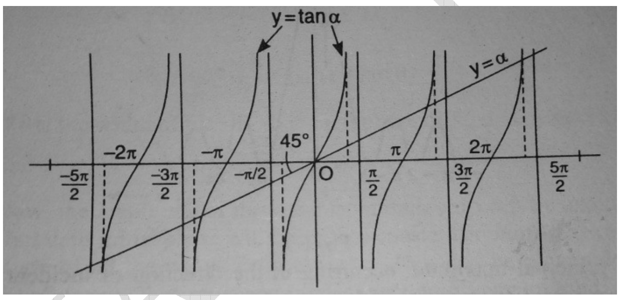
$$= A^2\left(\frac{1}{\alpha^2} 2\, Sin\,\alpha . Cos\,\alpha + \frac{Sin^2\,\alpha\, x -2}{\alpha^3}\right) = 0$$

$$\frac{dI}{d\alpha} = \frac{A^2}{\alpha^3}.\, 2\, Sin\,\alpha\,(\alpha . Cos\,\alpha - Sin\,\alpha) = 0$$

So, either $Sin\,\alpha = 0 \longrightarrow (1)$

Or $(\alpha . Cos\,\alpha - Sin\,\alpha) = 0 \longrightarrow (2)$

Eqn.(1) is the condition for minima. So, eqn.(2) should be the condition for secondary maxima. So, $\alpha . Cos\,\alpha = Sin\,\alpha$




Or $\alpha = \frac{Sin\,\alpha}{Cos\,\alpha}$

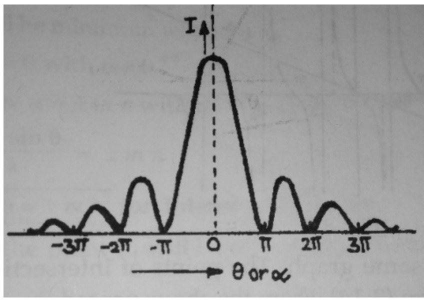
$$\boxed{\alpha = \tan\alpha}$$

In order to find out the solution for the above equation, two graphs are drawn by taking

$Y = \alpha \quad and \quad Y = \tan\alpha$

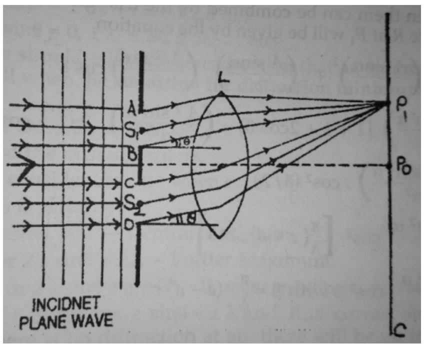
The points of intersection of the two graphs satisfy the above condition and these points will give the secondary maxima.

The right side figure gives the positions of maxima and minima and their intensities (I) with respect to the angle of diffraction (θ).




## Diffraction due to double slit

- If one of the two slits of double slit is closed, then only diffraction pattern is observed.
- If the two slits are open, then interference pattern also comes in to play.
- So, the combination of these two will give the intensity distribution of double slit.

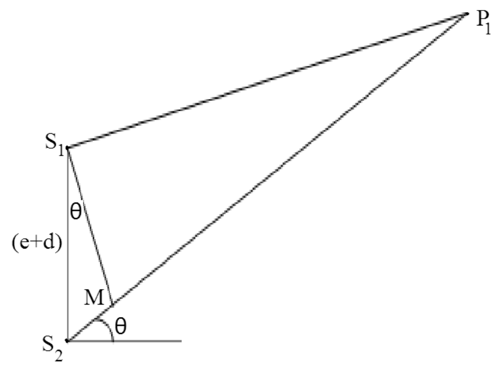



- In double slit, the two slits, AB & CD, have the same width 'e'. The space between the two slits, BC, is opaque and its width is 'd'. The centre points of the two slits are $S_1$ & $S_2$. Then the distance between $S_1$ & $S_2$ is

$$S_1 S_2 = \frac{e}{2} + d + \frac{e}{2} = (e + d)$$

- A parallel beam of light is incidenting on the double slit normally. Here the primary wave fronts are plane wave fronts. The secondary wave fronts or light rays from the two slits will progress
- The rays with out diffraction, refracted at the lens 'L' and form the image at the point $P_o$ on the screen. The screen is placed at the focal plane of the lens.
- The diffracted rays with angle 'θ' form the image at point $P_1$.
- The path difference or the phase difference between the rays from two slits will decide whether the point $P_1$ is bright or dark.

The path difference between the two rays from $S_1$ & $S_2$ to $P_1$ is $S_2M$. From the figure. $S_1M$ is the perpendicular drawn from $S_1$ on to $S_2P_1$.




From the $\Delta^{le}$ $S_1S_2M$ $\quad Sin\,\theta = \frac{S_2M}{S_1S_2} = \frac{S_2M}{(e+d)}$

∴The path difference $\quad S_2M = (e+d)\,Sin\,\theta$

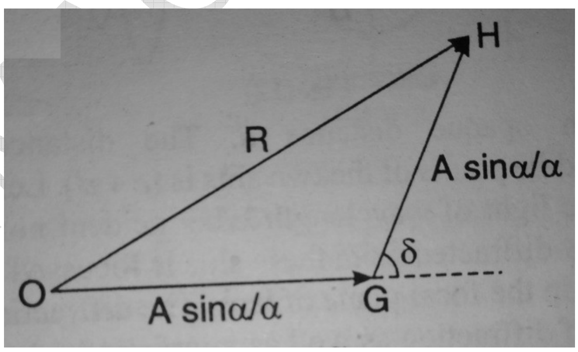
So, the phase difference is $\delta = \frac{2\pi}{\lambda}\,(S_2M)$

(or) $\quad \delta = \frac{2\pi}{\lambda}\,(e+d)\,Sin\,\theta$

The amplitudes of light coming from the two slits are equal

$R = \frac{A\,Sin\,\alpha}{\alpha}$ where

$$\boxed{\alpha = \frac{\pi}{\lambda}\,e\,Sin\,\theta}$$




- ✓ The vector sum of these two amplitudes will give the resultant amplitude at $P_1$.
- ✓ If the amplitudes of light coming from S1 and S2 are represented by OG, GH in the figure. Then OH gives the resultant amplitude.

Resultant intensity of light

$$I = \left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2 + \left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2 + 2\left(\frac{A\,Sin\,\alpha}{\alpha}\right)\left(\frac{A\,Sin\,\alpha}{\alpha}\right) Cos\,\delta$$

$$I = 2\left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2 + 2\left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2 Cos\,\delta = 2\left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2 (1 + Cos\,\delta)$$

$$I = 2\left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2 \left(2Cos^2\frac{\delta}{2}\right) \qquad \because (1 + Cos\,\delta) = 2Cos^2\frac{\delta}{2}$$

$$I = 4\left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2 \left(Cos^2\frac{\delta}{2}\right)$$

But $\delta = \frac{2\pi}{\lambda}(e+d)\,Sin\,\theta$ (or) $\frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\,Sin\,\theta,\quad Let\ \beta = \frac{\delta}{2}$

$$\therefore\ I = 4A^2\left(\frac{Sin\,\alpha}{\alpha}\right)^2 (Cos^2\beta)$$

Put $4A^2 = I_m \quad \Rightarrow \quad I = I_m\left(\frac{Sin\,\alpha}{\alpha}\right)^2 (Cos^2\beta)$

Here $\left(\frac{Sin\,\alpha}{\alpha}\right)^2$ will give the diffraction intensity pattern of single slit diffraction.

And $(Cos^2\,\beta)$ will give the <u>interference intensity pattern</u> of diffracted beam from two slits.

<u>Conditions for minima</u>

For minima I = 0 , then $\frac{Sin\,\alpha}{\alpha} = 0$ (or) $Cos\,\beta = 0$

$Sin\,\alpha = 0$ and $\alpha \neq 0$

$\alpha = \pm m\pi$ where m = 1, 2, 3…….

$\therefore \frac{\pi}{\lambda} e\,Sin\,\theta = \pm m\pi$ (or) $\boxed{e\,Sin\,\theta = \pm m\lambda}$

Similarly, if $Cos\,\beta = 0$ (or) $\beta = \pm(2n+1)\frac{\pi}{2}$

(or) $\frac{\pi}{\lambda}(e+d)\,Sin\,\theta = \pm(2n+1)\frac{\pi}{2}$

$\boxed{(e+d)\,Sin\,\theta = \pm(2n+1)\frac{\lambda}{2}}$ n = 0, 1, 2, 3, 4……..

The equations in the above two boxes are the conditions for minima.

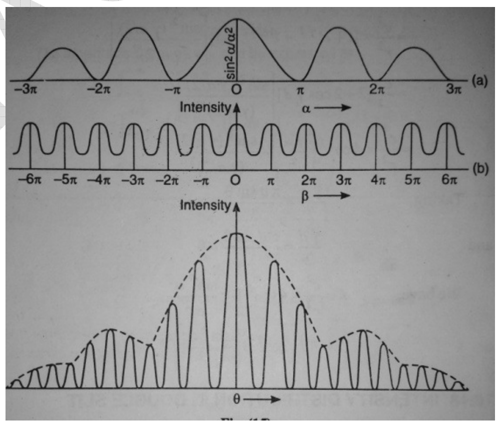
<u>Condition for maxima</u>

To get maxima

$Cos\,\beta = 1$ (or) $\beta = \pm n\pi$

(or) $\frac{\pi}{\lambda}(e+d)\,Sin\,\theta = \pm n\pi$

$\boxed{(e+d)\,Sin\,\theta = \pm n\lambda}$




☆ In the right side figure, the lower graph gives the intensity distribution due to diffraction pattern of one slit.

☆ The middle graph gives the intensity distribution due to interference pattern of two slit.

☆ The lower graph gives the intensity distribution of combined effect of diffraction and interference.

<u>Case 1</u> :-

If d = e then (d + e) = 2e

Then the condition for maxima

$(e+d)\,Sin\,\theta = \pm n\lambda$ becomes

$2e\,Sin\,\theta = \pm\lambda$

$2e\,Sin\,\theta = \pm 2\lambda$ (or) $e\,Sin\,\theta = \pm\lambda$ Condition for minima

$2e\,Sin\,\theta = \pm 3\lambda$

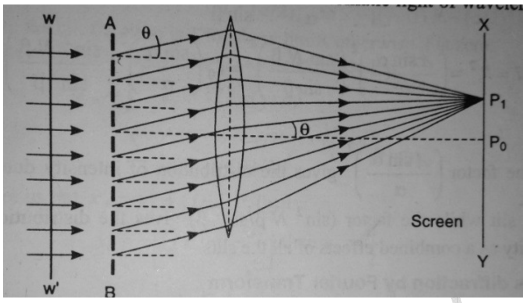
$2e\,Sin\,\theta = \pm 4\lambda$ (or) $e\,Sin\,\theta = \pm\lambda$ Condition for minima

…… etc.

⇨ The maxima will not appear at minima. So $n = \pm 2, \pm 4, \pm 6 \ldots..$ fringes will not appear and these are called missing order (or) missing maxima.

<u>Case 2</u> :- If d = 2e then, $n = \pm 3, \pm 6, \pm 9 \ldots..$ fringes will not appear and these are called missing order (or) missing maxima.

**Fraunhofer diffraction due to N-slits – plane diffraction grating**




- If a series of slits, straight and equal width, are arranged, the arrangement is called grating.
- The slit is transparent and having a width equal to 'e'.
- The distance between any two successive slits is 'd' and it is opaque. The value (e + d) is called grating element.
- A parallel beam of light is incidenting, normally, on the grating plate AB having N no. of slits.
- The rays with out diffraction refracted at the lens and form the image at the point $P_o$ on the screen. The screen is placed at the focal plane of the lens.
- The diffracted rays with angle θ form the image at point $P_1$.
- The path difference or the phase difference between the rays from slits will decide whether the point $P_1$ is bright or dark.
- Each slit in the grating acts as a single slit.

The amplitude of light coming from single slit $= \frac{A\,Sin\,\alpha}{\alpha}$ where $\boxed{\alpha = \frac{\pi}{\lambda}\,e\,Sin\,\theta}$

The path difference between the rays coming from two successive slits = $(e+d)Sin\,\theta$

Then, the phase difference = $\frac{2\pi}{\lambda}(e+d)\,Sin\,\theta$

Let $\frac{2\pi}{\lambda}(e+d)\,Sin\,\theta = 2\beta$

(or) $\boxed{\beta = \frac{\pi}{\lambda}(e+d)\,Sin\,\theta}$

We know the resultant of n simple harmonic motions or disturbances

$$R = \frac{a\,Sin\frac{nd}{2}}{Sin\frac{d}{2}} \longrightarrow (1)$$

Comparing the n simple harmonic motions with N slits

$$a = \frac{A\,Sin\,\alpha}{\alpha}\ ,\quad n = N \text{ and } d = 2\beta$$

Substituting these values in the eqn. (1)

Then, $$R = \frac{A\,Sin\,\alpha}{\alpha}\cdot\frac{SinN\beta}{Sin\,\beta}$$

∴ The intensity of light $I = R^2 = \left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2\cdot\left(\frac{SinN\beta}{Sin\,\beta}\right)^2$

Here $\left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2$ gives the intensity distribution in diffraction pattern due to single slit. This is called "shape factor".

And $\left(\frac{SinN\beta}{Sin\,\beta}\right)^2$ gives the intensity distribution in interference pattern due to N slits. This is called "grating factor".

Condition for principal maxima

If $\beta = \pm n\pi$

Then $Sin\,\beta = 0$ and $SinN\beta = 0$

So, $\frac{SinN\beta}{Sin\,\beta}$ will become indetermanent.

In order to solve this equation, Hospital's rule is applied.

According to Hospital's rule

$$\lim_{\beta\to\pm n\pi}\frac{SinN\beta}{Sin\,\beta} = \lim_{\beta\to\pm n\pi}\frac{\frac{d}{d\beta}(SinN\beta)}{\frac{d}{d\beta}(Sin\,\beta)}$$

$$= \lim_{\beta\to\pm n\pi}\frac{N\,Cos\,N\beta}{Cos\,\beta} = \pm N$$

So, intensity of light $I_{max} = \left(\frac{A\,Sin\,\alpha}{\alpha}\right)^2 N^2$

∴ The condition for maximum intensity is $\beta = \pm\, n\pi$

$$\frac{\pi}{\lambda}(e+d)\,Sin\,\theta = \pm\, n\pi \quad \text{(or)} \quad \boxed{(e+d)\,Sin\,\theta = \pm\, n\lambda}$$

This equation is called grating equation.

n = 0 is zeroth order

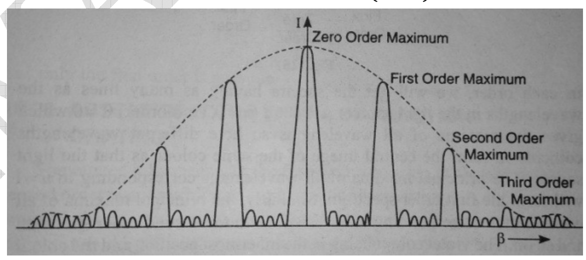
n = 1 is 1st order

n = 2 is 2nd order

As the equation has ± sign, there are two spectra on either side of zeroth maxima.

To find the position of the principal maxima, the equation is $Sin\,\theta = \pm\frac{n\lambda}{(e+d)}$

This equation is independent of N.




Condition for minima

For minima $\frac{SinN\beta}{Sin\,\beta} = 0$

i.e. $SinN\beta = 0 \quad and \quad Sin\,\beta \neq 0$

( If $Sin\,\beta = 0$ then it is the condition for principal maxima.)

$SinN\beta = 0$ (or) $\boxed{N\beta = \pm m\pi}$

$$\therefore \quad \beta = \pm\frac{m}{N}\pi$$

Note:- $\beta = 0, \pm\pi, \pm 2\pi, \pm 3\pi \ldots\ldots$ is the condition for maxima. So, at these places minima can not be located.

So, $\frac{m}{N} \neq 0, 1, 2, 3, 4 \ldots.$ (or) $m \neq 0, N, 2N, 3N, 4N \ldots.$

∴ $m = 1, 2, 3 \ldots\ldots (N-1), (N+1), (N+2) \ldots\ldots (2N-1), (2N+1) \ldots.$

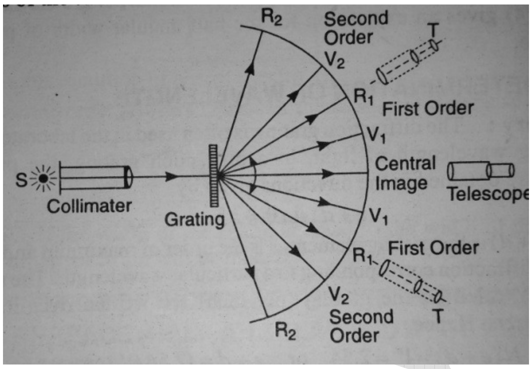
$$N\frac{\pi}{\lambda}(e+d)\,Sin\,\theta = \pm m\pi \quad \text{(or)} \quad \boxed{N(e+d)\,Sin\,\theta = \pm m\lambda}$$

This is the condition for minima.

Secondary maxima

There are (N-1) minima between two successive principal maxima and (N-2) maxima should be in between (N-1) minima. These maxima are called secondary maxima. That means the width of minima and secondary maxima decreases as N increases.

**Determination of wave length of light using grating – Normal incidence**




- In order to find out the wave length (λ) of light using spectrometer a plane diffraction grating can also be used instead of a prism.
- First the preliminary adjustments of the telescope, collimator and prism table are to be done.
- The source of light should be placed before collimator. The telescope should be placed in collinear with the collimator.

Normal incidence

- The circular scale is adjusted to $0^o$ -$0^o$ and $0^o$ – $180^o$.
- Now, the telescope is rotated with scale through $90^o$ and is made perpendicular to the collimator.
- Grating plate is now placed on the prism table. The prism table is rotated, with out disturbing the circular scale, such that the reflected image of the slit coincides with the vertical cross wire in the telescope.
- Now the telescope is rotated along with scale and made collinear with the collimator.
- Now the grating plate is making an angle $45^o$ with the incident light.
- The prism table along with the scale is rotated another $45^o$ and the grating plate is made perpendicular to the incident light.
- This is called "normal incidence".

The telescope is rotated and focused to the red colour in the 1st order spectrum on left side and the reading is taken as $R_{1Left}$. After that the telescope is turned to right side and focused to the red colour in the 1st order spectrum on right side and the reading is taken as $R_{1Right}$.

The difference $R_{1Right} \sim R_{1Left} = 2\theta$ (or) $\theta = \left(\frac{R_{1Right} \sim R_{1Left}}{2}\right)$

Here θ is the angle of diffraction for that colour.

For principal maxima, the grating equation gives $(e + d)\ Sin\ \theta = \pm n\lambda$

n= 1 the order of the spectrum.

$(e + d) = \frac{1}{N'}$ Here $N'$ = No. of lines (slits) per unit length on the grating plate

$\frac{Sin\ \theta}{N'} = n\lambda$ (or) $\boxed{\lambda = \frac{Sin\ \theta}{nN'}}$

The wave length λ of the light can be calculated by using this formula.

Similarly, the wave length λ can also be measured for other colours.

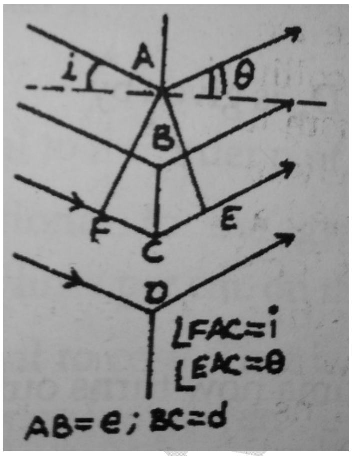
**Determination of wave length of light using grating –**
**Oblique incidence (or)minimum deviation.**

Consider a plane diffraction grating having slit width 'e' and opaque part width 'd'.

Then (e + d) = Grating element

Let the incident and diffracted rays make i and θ angles with the normal drawn to grating plate when the plate is in minimum deviation position.

To calculate the path difference between the rays passing through the points A & C, two normals, AF & AE, are drawn on to incident ray and diffracted ray.




The path difference Δ = FC + CE ⟶ (1)

From the $\Delta^{le}$ AFC $\quad Sin\, i = \frac{FC}{AC}$

$FC = AC\, Sin\, i = (e+d)\, Sin\, i$ ⟶ (2)

∵ AC = (e + d)

Similarly from the $\Delta^{le}$ AEC $\quad Sin\, \theta = \frac{CE}{AC}$

$CE = AC\, Sin\, \theta = (e+d)\, Sin\, \theta$ ⟶ (3)

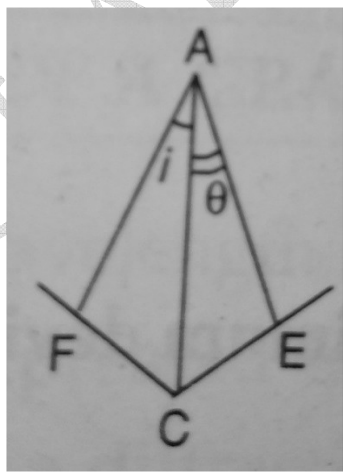
Substituting eqns. (2) & (3) in eqn. (1)

Then $\Delta = (e+d)\, Sin\, i + (e+d)\, Sin\, \theta$

$\Delta = (e+d)\, (Sin\, i + Sin\, \theta)$

For $n^{th}$ order principal maximum, the condition is

$$\boxed{(e+d)\, (Sin\, i + Sin\, \theta) = n\, \lambda} \longrightarrow (4)$$




But we know that

$$Sin\, A + Sin\, B = 2\, Sin\left(\frac{A+B}{2}\right).Cos\left(\frac{A-B}{2}\right)$$

As per this equation, the eqn. (4) becomes

$$(e+d)\, 2\, Sin\left(\frac{i+\theta}{2}\right).Cos\left(\frac{i-\theta}{2}\right) = n\, \lambda$$

$$\therefore \quad Sin\left(\frac{i+\theta}{2}\right) = \frac{n\, \lambda}{2\,(e+d)\, Cos\left(\frac{i-\theta}{2}\right)}$$

In order to get minimum value for $Sin\left(\frac{i+\theta}{2}\right)$, the value of $Cos\left(\frac{i-\theta}{2}\right)$ should be maximum.

$$Cos\left(\frac{i-\theta}{2}\right) = 1 \quad \text{(or)} \quad \frac{i-\theta}{2} = 0 \qquad \text{(or)} \quad i = \theta$$

Then $\quad i + \theta = 2\theta$

The minimum deviation angle $\quad D_m = 2\theta \quad$ (or) $\; i = \theta = \frac{D_m}{2}$

<u>Condition for central maximum</u>

$$(e+d)\left(Sin\, \frac{D_m}{2} + Sin\, \frac{D_m}{2}\right) = n\, \lambda$$

$$(e+d)\, 2\, Sin\, \frac{D_m}{2} = n\, \lambda$$

But $\quad (e+d) = \frac{1}{N'} \quad$ Here $N'$ = No. of lines (slits) per unit length on the grating plate

$$\therefore \quad \frac{2\, Sin\, \frac{D_m}{2}}{N'} = n\, \lambda \quad \text{(or)} \quad \boxed{\lambda = \frac{2\, Sin\left(\frac{D_m}{2}\right)}{nN'}}$$

From this equation, the value of the wave length of light $\lambda$ can be determined.

## Resolving power of grating

<u>Definition</u> :- The ability if the grating plate to distinguish the spectral lines of two wave lengths which are very close to each other.

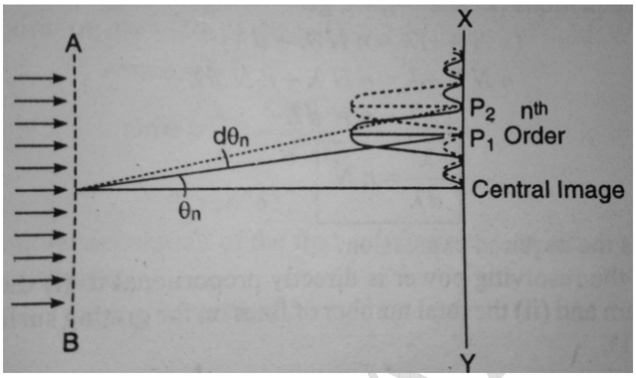
Resolving power $\boxed{R = \frac{\lambda}{d\lambda}}$

Here $\lambda = \frac{\lambda_1 + \lambda_2}{2}$ = Average of the two wave lengths $\lambda_1$ and $\lambda_2$

And $d\lambda = \lambda_1 \sim \lambda_2$ = Difference of the two wave lengths $\lambda_1$ and $\lambda_2$

<u>Explanation</u> :-

- A parallel beam of light is incidenting normally on a plane diffraction grating plate having 'N' slits.
- There are two wave lengths in the incident light and these are very close to each other.




Let the wave lengths be

$\lambda_1 = \lambda \quad and \quad \lambda_2 = \lambda + d\lambda$

The wave length λ is diffracted with angle $\theta_n$ and formed $n^{th}$ maximum at $P_1$.

$$(e + d)\, Sin\, \theta_n = n\, \lambda \longrightarrow (1)$$

Similarly, the wave length $\lambda + d\lambda$ is diffracted with angle $(\theta_n + d\theta)$ and formed its $n^{th}$ maximum at $P_2$.

$$(e + d)\, Sin\, (\theta_n + d\theta) = n(\lambda + d\lambda) \longrightarrow (2)$$

As per the Rayleigh's condition,

The two lights can be distinguished, only when the $n^{th}$ order principal maximum of $(\lambda + d\lambda)$ should be formed at the $n^{th}$ order $1^{st}$ minimum position of $\lambda$, i.e. at position $P_2$.

If the maxima are more close, then these two cannot be identified separately.

The condition for $n^{th}$ order $1^{st}$ minimum position of $\lambda$ at $P_2$ is

$$N(e + d)\, Sin\, (\theta_n + d\theta) = \pm\, m\lambda$$

Here $m \neq 0, N, 2N, 3n \ldots\ldots$

For $n^{th}$ order $1^{st}$ minimum, m = (Nn + 1)

$$N(e + d)\, Sin\, (\theta_n + d\theta) = \pm\, (Nn + 1)\lambda \longrightarrow (3)$$

Multiplying eqn. (2) by N,

$$N(e + d)\, Sin\, (\theta_n + d\theta) = Nn(\lambda + d\lambda) \longrightarrow (4)$$

The LHS of eqns. (3) & (4) are equal. So, their RHS are also equal.

$$\therefore\ (Nn + 1)\lambda = Nn(\lambda + d\lambda)$$

$$\cancel{Nn\lambda} + \lambda = \cancel{Nn\lambda} + Nn\, d\lambda$$

$$\therefore\ \frac{\lambda}{d\lambda} = Nn$$

So, the resolving power $\boxed{R = \frac{\lambda}{d\lambda} = Nn}$

The resolving power does not depend on the grating element (e + d).

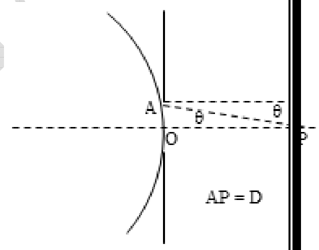
The resolving power depends on the total no. of lines on the grating plate 'N' and on the order of the spectrum 'n'.

# Chapter – V

# Diffraction – 2 (Fresnel's diffraction)

## Huygen – Fresnel theory

- As per Huygen's theory, light generated from a source will travel in Ether medium, as mechanical wave.
- These waves travel in different directions. The locus of all the points in the waves, which are in phase is called wave front (or) primary wave front.
- If the source is at finite distance then the wave front is in spherical shape and is called spherical wave front.
- If the source is at infinite distance then the wave front is a plane and is called plane wave front.
- Each and every point on the primary wave front will act as a source of light. This is called secondary source of light. Secondary source of light produces secondary wave fronts.
- These secondary wave fronts interfere with one another.
- In order to find out the intensity of light at a point Fresnel divided the primary wave front in to zones. These zones are called Fresnel's half period zones.




- Let the amplitude of light from a zone at the point P be $a_n$.

$a_n \propto A$ A = Area of the Fresnel's half period zone

$\propto \frac{1}{D}$ D = Distance between the point P and the Fresnel's half period zone

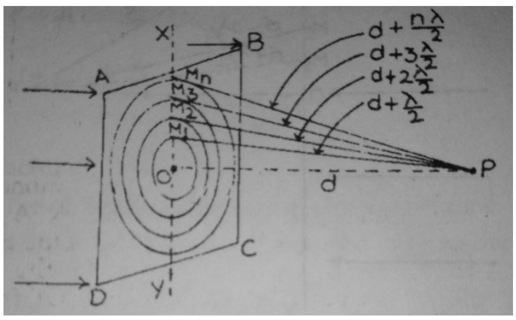
$\propto (1 + cos\theta)$ (1 + cos θ) = Obliquity factor & θ = Angle of deviation

(or) $\boxed{a_n \propto \frac{A\,(1+cos\theta)}{D}}$

If θ = 180°, then Cos 180° = -1 So, (1 + cos θ) = (1-1) = 0 and $a_n = 0$

∴ The secondary wave fronts will travel only in the for ward direction and not in the back ward direction, as the amplitude ($a_n$) in the back ward direction is zero.

## Fresnel's half period zones




- ☆ ABCD is the plane wave front of a monochromatic wave length 'λ' . Let 'P' be the point where the amplitude or the intensity of light is to be measured. This point P is at a perpendicular distance 'd' from the plane wave front.
- ☆ If different spheres of radii (d + λ/2), (d + 2λ/2), (d + 3λ/2) …. (d + nλ/2) are drawn, by taking P as the centre, then the cross-section of the plane wave front and the spheres will give the concentric circles of radii $OM_1$, $OM_2$ ….. $OM_n$ etc.

☆ The area between two successive circles is called zone. The path difference between two successive zones from the point P is λ/2 and phase difference is π and the time difference is T/2. So, these zones are called half period zones.

As the order of half period zone increases the deviation '$\theta_n$' increases and the obliquity factor value decreases.

Area of the half period zone :-

Area of 1st half period zone of radius $OM_1$ is

$$\pi\,(OM_1)^2 = \pi\,[(PM_1)^2 - (OP)^2]$$

$$\pi\,(OM_1)^2 = \pi\,[\left(d + \frac{\lambda}{2}\right)^2 - (d)^2]$$

$$\pi\,(OM_1)^2 = \pi[d^2 + \frac{\lambda^2}{4} + 2d\frac{\lambda}{2} - d^2]$$

Area of 1st zone $\boxed{\pi\,(OM_1)^2 = \pi d\lambda}$ $\because \frac{\lambda^2}{4}$ = Very small and can be neglected.

∴ Radius of the 1st circle $\boxed{OM_1 = \sqrt{d\lambda}}$

Area of the circle having radius $OM_2$ is

$$\pi\,(OM_2)^2 = \pi\,[(PM_2)^2 - (OP)^2]$$

$$\pi\,(OM_2)^2 = \pi\,[(d + \lambda)^2 - (d)^2]$$

$$\pi\,(OM_2)^2 = \pi[d^2 + \lambda^2 + 2d\lambda - d^2]$$

Area of 2nd circle $\boxed{\pi\,(OM_2)^2 = 2\pi d\lambda}$ $\because \lambda^2$ = Very small and can be neglected.

∴ Radius of the 2nd circle $\boxed{OM_2 = \sqrt{2d\lambda}}$

Area of the 2nd zone = Area of 2nd circle – Area of 1st circle

$$\pi\,(OM_2)^2 - \pi\,(OM_1)^2 = 2\pi d\lambda - \pi d\lambda = \pi d\lambda$$

Area of the 1st zone = Area of the 2nd zone = $\pi d\lambda$

So, *all the zones have equal areas* $= \pi d\lambda$ ⟶ (1)

The radii of the circles are $OM_1 = \sqrt{1.d\lambda}$

$OM_2 = \sqrt{2.d\lambda}$ …………

$OM_n = \sqrt{n.d\lambda}$

(or) $OM_n = \sqrt{n}$

i.e. The radii of the circles are directly proportional to the square root of natural nos.

Average distance of the zone :-

The nth zone is in between nth and (n-1)th circles.

Distance of nth circle from the point 'P' $= d + n\,\frac{\lambda}{2}$

Distance of (n-1)th circle from the point 'P' $= d + (n-1)\,\frac{\lambda}{2}$

Average distance of nth zone from the point P is $D = \frac{\left[d + n\frac{\lambda}{2}\right] + [d + (n-1)\frac{\lambda}{2}]}{2}$

∴ $\boxed{D = d + (2n-1)\frac{\lambda}{4}}$ ⟶ (2)

Obliquity factor :-

As the order of the zone increases. The deviation "θ" value increases. The obliquity factor (1 + cos θ) value decreases.

But we know that $a_n \propto \frac{A\,(1+cos\theta)}{D}$

Substituting eqns. 1) and 2) in this equation

$$\boxed{a_n \propto \frac{\pi d\lambda}{d+(2n-1)\frac{\lambda}{4}}\, x(1+cos\theta)} \longrightarrow (3)$$

This equation says that as the order 'n' of the zone increases the amplitude ($a_n$) of the light from that zone decreases.

<u>Relative phase of the half period zones</u> :-

The path difference between any two successive circles is λ/2 and the phase difference is π.

The phases of 0th and 1st circles are 0, π. The average phase of 1st zone = π/2

Similarly, the phases of 1st and 2nd circle are π, 2 π. The average phase of 2nd zone = $\frac{\pi+2\pi}{2} = \frac{3\pi}{2}$

So, the average phases of successive zones are $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $etc.$

The phase differences between successive zones $= \frac{3\pi}{2} - \frac{\pi}{2} = \frac{5\pi}{2} - \frac{3\pi}{2} = \frac{7\pi}{2} - \frac{5\pi}{2} = \pi$

i.e. <u>The successive zones have a phase difference π and the amplitudes are in opposite directions.</u>

If the amplitudes of light from the successive zones are $a_1, a_2, a_3, a_4, a_5$ ……

The resultant amplitude $a = a_1 - a_2 + a_3 - a_4 + a_5$ …… ⟶ (4)

From the equation (3)

$a_1 > a_2 > a_3 > a_4 > a_5$ ……

As the amplitudes are decreasing, the amplitude of light from a zone can be taken as the average of amplitudes due to preceding zone and succeeding zone.

i.e. $a_2 = \frac{a_1+a_3}{2}$ $a_4 = \frac{a_3+a_5}{2}$ …. ⟶ (5)

From equations (4) & (5) the resultant amplitude is

$$a = \frac{a_1}{2} + \left(\frac{a_1}{2} - a_2 + \frac{a_3}{2}\right) + \left(\frac{a_3}{2} - a_4 + \frac{a_5}{2}\right) + \frac{a_5}{2} \ldots$$

$\therefore a = \frac{a_1}{2} + \frac{a_n}{2}$ If n = odd

$a = \frac{a_1}{2} + \frac{a_{n-1}}{2} - a_n$ If n = even

As n value is large, values $a_{n-1}, a_n$ values are very small and can be neglected.

Then $\boxed{a = \frac{a_1}{2} \;\; or \;\; a^2 = \frac{a_1^2}{4} \qquad i.e. \qquad I = \frac{I_1}{4}}$

Here I = Resultant intensity of light due to the wave front or due to all zones

$I_1$= Intensity of light due to the 1st zone.

* The amplitude of light from the whole wave front is equal to half of the amplitude of light coming from the 1st half period zone.

(or)

* The intensity of light from the whole wave front is equal to one fourth of the intensity of light coming from the 1st half period zone.

**<u>Rectilinear propagation of light</u>**

☆ From the above discussion, it is clear that, the intensity of light due to the whole wave front at point P, is ¼ th that due to the 1st half period zone.

- ☆ In the total area of the wave front, the effective area is equal to half of the area of the 1st half period zone only.
- ☆ If the size or the area of the obstacle is greater than half of the area of the 1st half period zone then the intensity at P is zero (dark). Then diffraction phenomenon (or bending of light) is said to be not observed. i.e. The light takes rectilinear propagation.
- ☆ If the size or the area of the obstacle is less than half of the area of the 1st half period zone then the intensity at P is not zero. Then diffraction phenomenon (or bending of light) is said to be observed.

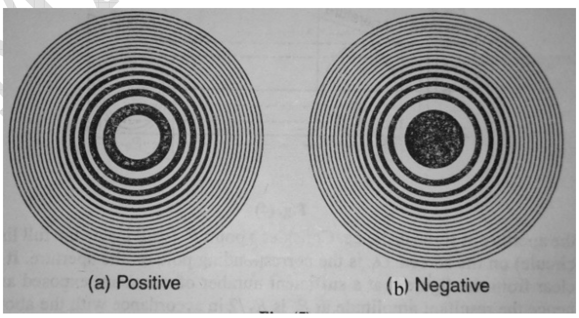
**Zone plate**

Construction:-

- ✪ In Fresnel's half period zones, the radii of the circles are directly proportional to the square root of the natural numbers. $r_n \propto \sqrt{n}$
- ✪ Concentric circles are drawn on a white drawing paper such that their radii are equal to the square root of the natural numbers.
- ✪ The alternate zones are painted with black colour and these zones act as opaque.
- ✪ This drawing paper is photographed and the developed negative is called zone plate.
- ✪ The zones painted black on drawing paper will become transparent in the zone plate and vice-versa.

Zone plates are of two types.

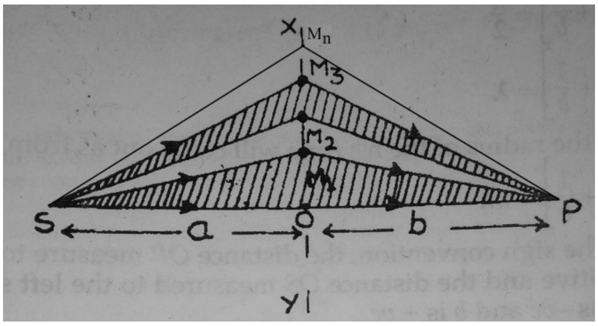
1. Positive zone plate 2. Negative zone plate




Positive zone plate :- If odd zones are transparent and even zones are opaque, then that zone plate is called positive zone plate (Fig. a).

Negative zone plate :- If odd zones are opaque and even zones are transparent, then that zone plate is called negative zone plate(Fig. b).

Working of zone plate :-




- ☞ Zone plate is the device to verify the working of Fresnel's half period zones.
- ☞ 'S' is the source of light and it is at a distance 'a' from the centre of the zone plate 'O'. 'P' is the point on the screen and it is at a distance 'b' from the point 'O'.
- ☞ The radii of Fresnel's half period circles are $OM_1 = r_1$, $OM_2 = r_2$ . . . . $OM_n = r_n$
- ☞ The path difference between the light rays passing through two successive half period circles is $\lambda/2$.

$$SO = a \quad \text{and} \quad OP = b$$

$$SM_1 + M_1P = a + b + \left(\frac{\lambda}{2}\right)$$

$$SM_2 + M_2P = a + b + 2(\frac{\lambda}{2})$$

. . . . . . . . . . . . . . . . .

$$SM_n + M_nP = a + b + n(\frac{\lambda}{2}) \longrightarrow (1)$$

But from the $\Delta^{le}$ $SM_nO$
$$SM_n^2 = SO^2 + OM_n^2$$
$$SM_n^2 = a^2 + r_n^2$$
$$SM_n = (a^2 + r_n^2)^{1/2}$$
$$SM_n = a(1 + \frac{r_n^2}{a^2})^{1/2}$$

By using Binomial theorem $SM_n = a(1 + \frac{1}{2} . \frac{r_n^2}{a^2})$

$$\therefore \quad SM_n = (a + \frac{1}{2} . \frac{r_n^2}{a}) \longrightarrow (2)$$

Similarly, from the $\Delta^{le}$ $PM_nO$ $M_nP = (b + \frac{1}{2} . \frac{r_n^2}{b}) \longrightarrow (3)$

Substituting eqns. (2) and (3) in eqn. (1) ,

Then
$$(a + \frac{1}{2} . \frac{r_n^2}{a}) + (b + \frac{1}{2} . \frac{r_n^2}{b}) = a + b + n(\frac{\lambda}{2})$$
$$(\frac{1}{2} . \frac{r_n^2}{a}) + (\frac{1}{2} . \frac{r_n^2}{b}) = n(\frac{\lambda}{2})$$

(or)
$$r_n^2 \left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$
$$\therefore \left(\frac{1}{a} + \frac{1}{b}\right) = \frac{n\lambda}{r_n^2}$$

By considering the sign convention
$$\boxed{\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{n\lambda}{r_n^2}}$$

This equation is similar to $\left(\frac{1}{v} - \frac{1}{u}\right) = \frac{1}{f}$ & this equation is for the focal length of a convex lens.

Comparing these two eqns. $\boxed{\frac{1}{f} = \frac{n\lambda}{r_n^2}}$ ( or ) $\boxed{f = \frac{r_n^2}{n\lambda}}$

So, the zone plate works as a convex lens.

**Phase reversal zone plate**

- In case of wave front, all the zones are transparent. The amplitudes of light from alternative zones are positive and the amplitudes of light from other zones are negative.
- Amplitudes of light coming from successive zones of the wave front are $a_1$, $-a_2$, $a_3$, $-a_4$ ….
- The resultant amplitude of light from the total wave front is $\frac{a_1}{2}$ or the intensity of the total wave front is $I = \frac{a_1^2}{4}$. Here $a_1$ = amplitude of the light from the 1st zone.
- In case of zone plate, alternate zones are transparent and the other zone are made opaque.
- Amplitudes of light coming from transparent zones of the zone plate are $a_1$, $a_3$, $a_5$ ….
- So, the amplitude and intensity of light from the zone plate is very high than those from the total wave front.

- If the zones having negative amplitudes in the zone plate are not eliminated and more over their phases are reversed by introducing a phase change of π, then that process is called phase reversal.
- This zone plate is called phase reversal zone plate.
- The resultant amplitude of light coming from phase reversal zone plate is ( $a_1+a_2+a_3+a_4$ …. ).
- The resultant amplitude of light from phase reversal zone plate is twice to that from the zone plate.

- Resultant amplitude of light from wave front $=a_1 - a_2 + a_3 - a_4 + a_5 - a_6$ ….
- Resultant amplitude of light from zone plate = $a_1 + a_3 + a_5$ ….
- Resultant amplitude of light from phase reversal zone plate = $a_1 + a_2 + a_3 + a_4 + a_5 + a_6$ ….

**Preparation of phase reversal zone plate**

✓ The alternate Fresnel's half period zones are made transparent.

✓ The other alternate zones are coated with a transparent layer to produce an additional path difference of λ/2 or a phase difference of π of the incident wave length.

✓ This is called phase reversal zone plate.

Preparation

* A clean glass plate is coated with gelatin.
* This coating is sensitized by immersing the plate in weak potassium dichromate and dried in the dark.
* This plate is placed in contact with the zone plate and exposed to Sun light.
* Light passing through transparent zones acts on gelatin layer and makes the layer insoluble in water.
* Gelatin layer in contact with opaque zones was made soluble in water.
* The glass plate is immersed in water. Then the gelatin layers of unexposed zones are dissolved. This is the phase reversal zone plate.

**Similarities between zone plate and convex lens**

| S.No. | Zone plate | Convex lens |
|---|---|---|
| 1. | The focal length of zone plate is $\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$ | The focal length of convex lens is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ |
| 2. | On the other side of the zone plate real image is formed. | On the other side of the convex lens real image is formed. |
| 3. | $f_n = \frac{r_n^2}{n\lambda}$ | $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$ |
| 4. | Different colours have different foci as the focal length depends on λ | Different colours have different foci as the focal length depends on μ |
| 5. | This has chromatic aberration. | This has chromatic aberration. |



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**Differences between zone plate and convex lens**

| S.No. | Zone plate | Convex lens |
|---|---|---|
| 1. | Image is formed due to diffraction. | Image is formed due to refraction. |
| 2. | The zone plate acts as converging lens as well as diverging lens simultaneously. | The convex lens acts as converging lens only. |
| 3. | One colour has different focal lengths $f_n = \frac{r_n^2}{n\lambda}$. One colour has different $r_n$ values. | One colour has one focal length $\frac{1}{f} = (\mu - 1)(\frac{1}{R_1} - \frac{1}{R_2})$ . One colour has one μ. |
| 4. | In case of zone plate the focal length of violet is greater than the focal length of red. ($f_V > f_R$) | In case of convex lens the focal length of red is greater than the focal length of violet. ($f_R > f_V$) |
| 5. | The image formed due to zone plate has less intense. | The image formed due to convex lens has more intense. |
| 6. | In case of zone plate the path difference between the light rays coming from different transparent zones is nλ. | In case of convex lens, all the rays that form the image have same optical path i.e. path difference is zero. |
| 7. | In case of zone plate the images are formed with X-rays and micro waves. | In case of convex lens images are formed due to the visible light only. |

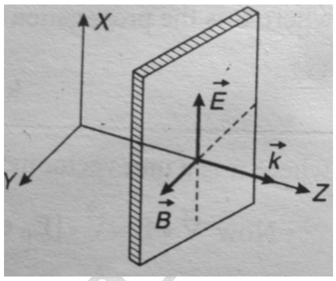
**Differences between diffraction and interference.**

| S.No. | Diffraction | Interference |
|---|---|---|
| 1. | In case of diffraction, the secondary wave fronts coming from different points of the same primary wave front are superimposed. | In case of interference, two primary wave fronts coming from two coherent sources are superimposed. |
| 2. | The regions of minimum intensity are not completely dark. | If the amplitudes of interfering waves are equal then regions of minimum intensity (dark band) are perfectly dark. |
| 3. | In case of diffraction the fringe widths are not equal. | In case of interference the fringe widths mat or may not be equal. |
| 4. | The central maximum has highest intensity and the intensity goes on decreasing as moved away from the centre. | All the maxima have the same intensity in interference. |

# Chapter – VI

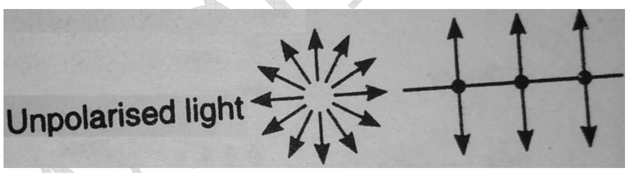
## Polarisation

- The properties of light, interference and diffraction, can be explained on the basis of wave theory of light that means the wave maybe longitudinal or transverse.
- But to explain polarisation, the light wave must be transverse wave.
- As per Maxwell's theory, light is electromagnetic wave. As per this theory, in the electromagnetic wave, electric field and magnetic field are perpendicular to each other and their vibrations are perpendicular to the propagation of the wave.

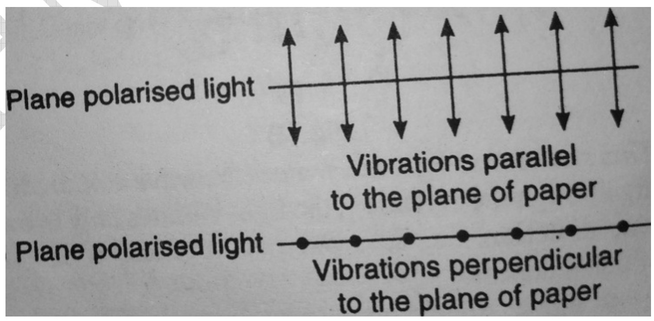



- Electric field shows more influence on the eye but magnetic field shows very little influence on the eye. So, electric field waves are taken as light waves.

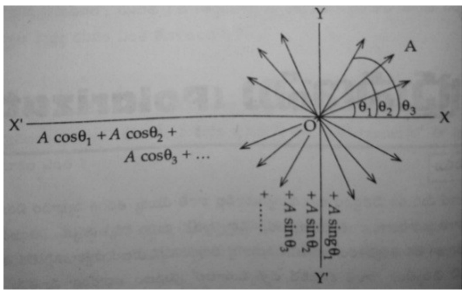
**Unpolarised light** :- If the vibrations of light waves are in different planes and they are perpendicular to the propagation of light wave and symmetric about the propagation of light, then that light is called 'unpolarised light'. The unpolarised light is as shown in the figure.




**Polarised light** :- If the vibrations of light wave are confined to a single plane, that light is called 'polarized light' (or) 'plane polarised light' (or) 'linearly polarised light'.
The plane polarised light is as shown in the figure.




**σ,π - components** :- Let an unpolarised light is travelling perpendicular to the plane of the paper and the vibrations of the light Waves are making angles $\theta_1$, $\theta_2$, $\theta_3$ . . . . etc. with X - axis and all of them are having amplitude 'A'.

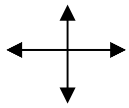



Then

Horizontal component $= A\cos\theta_1 + A\cos\theta_2 + \ldots$

Vertical component $= A\sin\theta_1 + A\sin\theta_2 + \ldots$

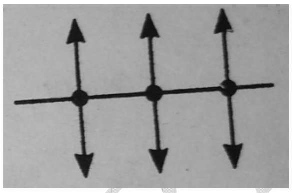
These two components can be shown as follows



$A\cos\theta_1 + A\cos\theta_2 + \ldots$

$A\sin\theta_1 + A\sin\theta_2 + \ldots$

If the propagation of light is in the Plane of the paper then the above two components can be shown as in figure.



π - component :- If the vibrations of the Waves of light are in the Plane of the paper then those components are called π – components and these are shown as ( ↕ ) .

**σ -** component :- If the vibrations of the Waves of light are perpendicular to the plane of the paper then those components are called σ – components and these are shown as ( ● ) .

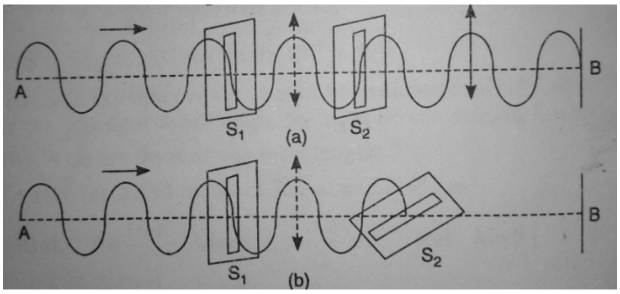
**Unpolarised light** :- If the intensities of π - components and σ - components are equal in the propagated light then that light is called "unpolarised light"

**Plane polarized light (or) Linearly polarized light** :- If the light consists of either π - component or σ – component that light is called Plane polarized light (or) Linearly polarized light.

**Partially polarised light** :- If the intensities of π – components and σ – components are not equal in the light that light is called partially polarised light.

## Polarization – Explanation experiments

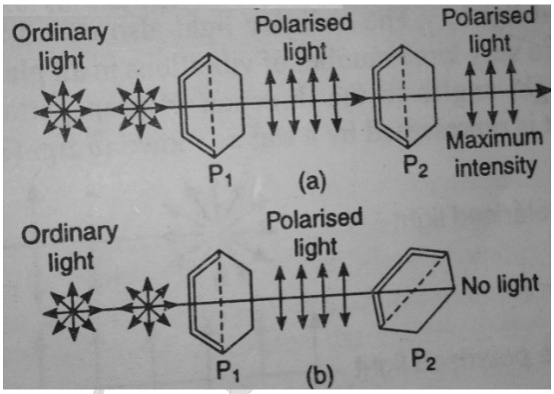
## Experiment 1 :-




- Consider a rope AB, passed through two parallel slits $S_1$ and $S_2$ of two frames and second end B of the rope is tied to the fixed point.
- Now vibrate the point A of the rope, perpendicular to the length of the rope and parallel to the slit $S_1$. Then the transverse waves generated in the rope travel through $S_1$ and $S_2$ and reach the point B, **Fig. (a).**
- Now rotate the slit $S_2$ in the same plane through $90^0$ and is made perpendicular to the slit $S_1$. Vibrate the point A, as in the previous case, i.e. perpendicular to the length of the rope and parallel to the slit $S_1$. Then the transverse waves generated in the rope travel through $S_1$ and reaches $S_2$ and does not go beyond $S_2$, **Fig. (b).**
- Now replace the rope AB by a spring and the point A of the spring is vibrated along the length of it. Then longitudinal waves are generated in the spring and these waves reach the point B in the above two cases i.e. when the two slits $S_1$ and $S_2$ are parallel and perpendicular positions.

- It means that if the wave is transverse then it travels through the two slits $S_1$ and $S_2$ when they are parallel. If the slits are perpendicular then the slit $S_2$ can not allow the wave to pass through it.
- If the wave is longitudinal then it can travel through the two slits $S_1$ and $S_2$ and reaches point B what ever the angle it may be between the slits.

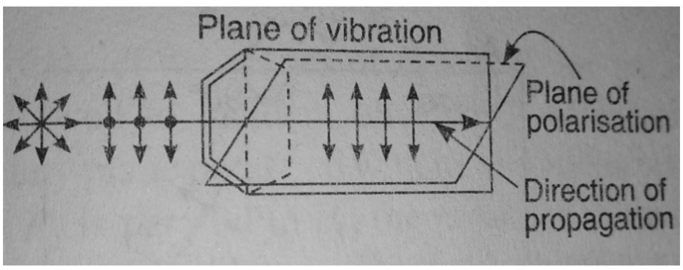
**Experiment 2**:-




- This experiment is also similar to that of the above experiment. The two frames are replaced by two tourmaline crystals $P_1$ and $P_2$. The principal axes of the two crystals acts similar to that of the two slits $S_1$ and $S_2$.
- The two tourmaline crystals $P_1$ and $P_2$ are placed on a common axis such that their planes are perpendicular to the propagation of the light and their principal axes are parallel to each other.
- Before the crystal $P_1$ unpolarised light source is placed and after the crystal $P_2$ the screen is placed.
- Now the polarised light reaches on to the screen. Now if the crystal $P_2$ is rotated about the direction of propagation of light, then the intensity of light on the screen decreases gradually and becomes zero when the two crystal principal axes are mutually perpendicular.
- If the crystal $P_2$ is rotated another $90^0$ then the two axes of $P_1$& $P_2$ once again become parallel and the intensity of light on the screen increases to the initial value.
- As the intensity of light on the screen varies on rotating the crystal $P_2$, we can say that the light is transverse wave. If light is longitudinal wave then there would not be any change in the intensity.
- So, light has transverse wave nature.

## Plane of vibration and plane of polarization :-

**Plane of vibration** :- The plane in which the vibrations of plane polarized light takes place is called plane of vibration.




**Plane of polarization** :- The plane in which the vibrations of plane polarized light do not take place is called plane of polarisation.

- The plane of vibration and the plane of polarization are perpendicular each other.

## Malus law

- When an unpolarised light incident on a tourmaline crystal then polarized light emerges out from it. So, this crystal is called "polariser". This polarised light can be tested by sending it through another tourmaline crystal, then this crystal is called "analyser".
- Let "a" be the amplitude and "$I_o$" be the intensity of the plane polarized light coming out from the polariser. Then $I_o = a^2 \longrightarrow$ (1)
- Let "$a_\theta$" be the amplitude and "$I_\theta$" be the intensity of the plane polarized light coming out from the analyser when the angle between the axes of the polarizer and analyser is θ.

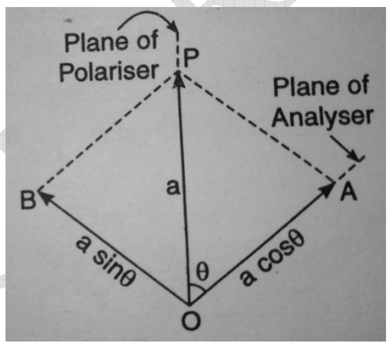
$$I_\theta = a_\theta^2 \longrightarrow \textbf{(2)}$$

But $a_\theta = a\cos\theta$

(or) $a_\theta^2 = a^2 \cos^2\theta$

From eqns. (1) & (2) $\therefore I_\theta = I_o \cos^2\theta$

(or) $\boxed{I_\theta \propto \cos^2\theta}$




**Definition** :- The intensity of polarized light emerging out from the analyser $I_\theta$, is directly proportional to square of the cosine of the angle $\theta$ between the axes of the polariser and analyser.

1. If the angle between the axes of the polariser and analyser is 0°, then $\theta = 0$

   $\cos 0 = 1$ then $I_\theta = I_o \cos^2 0$

   $I_\theta = I_o$

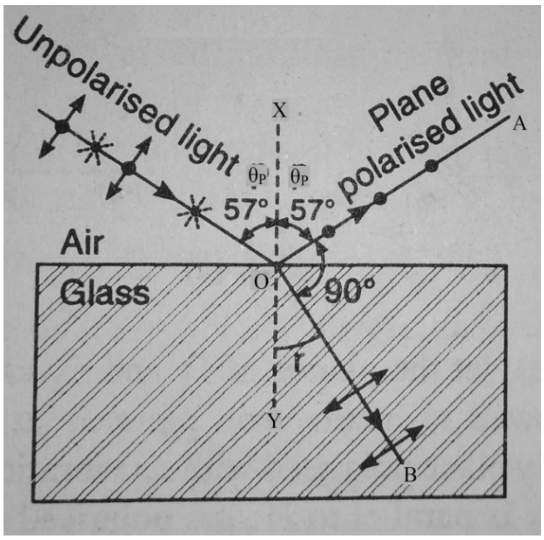
2. If the angle between the axes of the polariser and analyser is 90°, then $\theta = 90^0$

   $\cos 90^0 = 0$ then $I_\theta = 0$

   $I_\theta = 0$

## Polarization due to reflection – Brewster's law

- When unpolarised light incident on a glass slab, the reflected light is partially polarised light. This was first detected by Malus.
- The percent (or) degree of polarization changes with the angle of incident.
- At a particular angle of incidence ($\theta_P$) the reflected light is completely "plane polarised light".
- **Angle of polarization ($\theta_P$)** :- The angle of incidence of unpolarised light for which the reflected light is completely polarised is called angle of polarization (or) Brewster's angle.



- The value of angle of polarization ($\theta_P$) depends on the nature of the glass slab on which the light incident (or) the refractive index and also on the wave length light that incident.
- **Brewster's law** :- The tangent of the angle of polarization ($\theta_P$) is numerically equal to the refractive index of the glass slab and at this angle, the reflected and refracted rays are perpendicular to each other.

$$\boxed{\tan\theta_P = \mu}$$

Unpolarised light incident at point "O" with an angle of incident equal to the angle of polarization $\theta_P$ and reflected as OA with the same angle. This light is completely polarised light. But the refracted light ray 'OB' with angle of refraction (r) is partially polarised light.

From Snell's law $\mu = \frac{\sin i}{\sin r}$

$$\mu = \frac{\sin\theta_P}{\sin r} \longrightarrow (1) \qquad \because i = \theta_P$$

From Brewster's law $\mu = \tan\theta_P$

$$\mu = \frac{\sin\theta_P}{\cos\theta_P} \longrightarrow (2)$$

Comparing the above two equations $\sin r = \cos\theta_P$

(or) $\cos(90^o - r) = \cos\theta_P$ (or) $(90^o - r) = \theta_P$

$$r + \theta_P = 90^o \longrightarrow (3)$$

From the figure and the equation (3)

$$\angle XOA + \angle YOB = 90^o$$

But $\angle XOY = 180^o$

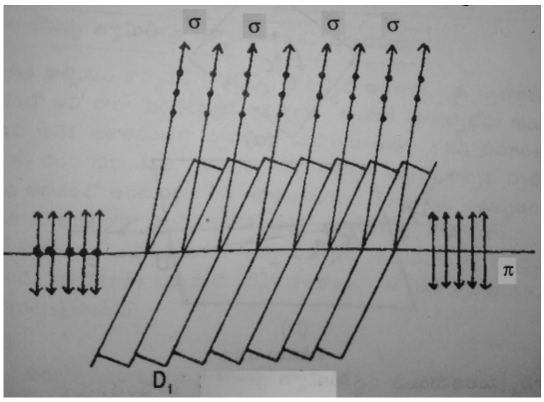
$$\angle XOY = \angle XOA + \angle AOB + \angle YOB = 180^o$$

$$\angle AOB + 90^o = 180^o \quad \because \angle XOA + \angle YOB = 90^o$$

$$\therefore \angle AOB = 90^o$$

So, $OA, OB$ are perpendicular. i.e. the reflected and refracted rays are perpendicular to each other.

## Polarization due to refraction – Pile of plates

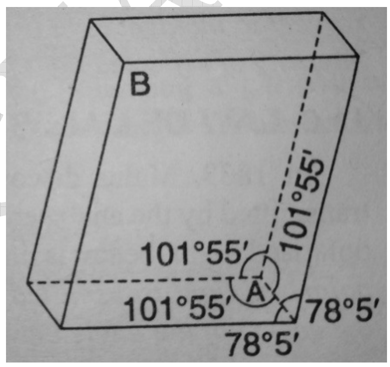



- The arrangement, to get polarised light due to refraction, consists of 15 to 20 similar glass plates. These glass plates are arranged parallel to one another. The unpolarised light incident on the 1st glass plate with angle of polarisation ($\theta_P$).
- A part of the ray is reflected, the reflected ray is plane polarised and it consists of 15% of σ – components only.
- The remaining part refracts & it is partially polarised light. This part consists of 85 % σ – components and 100% π –components.

- This part of light also incident on the 2nd glass plate with same angle ($\theta_P$). In this 15% of σ – components are reflected. So, 15% of σ – components decrease in refracted light and no decrease in π –components.
- Like this, as 15% of σ – components are reflected at each reflection, the σ – components decrease gradually in refracted light and finally becomes zero. But only π –components remain in the final refracted light. This is also plane polarised light.
- In this manner the polarised light can be got by refraction method also.

## Geometry of Calcite crystal

- Calcite crystal is hydrated Calcium carbonate (Ca $CO_3$). This belongs to the rhombo-hedral class of hexogonal system. This has six faces and each face is rhombus. In each face the opposite angles are $101^o\ 55^1$ and $78^o\ 5^1$ .
- The calcite crystal has two opposite corners A and B where three abtuse angles ($101^o\ 55^1$) meet. These two corners are called "blunt corners". At all other six corners, the combination of acute and abtuse angles take place.



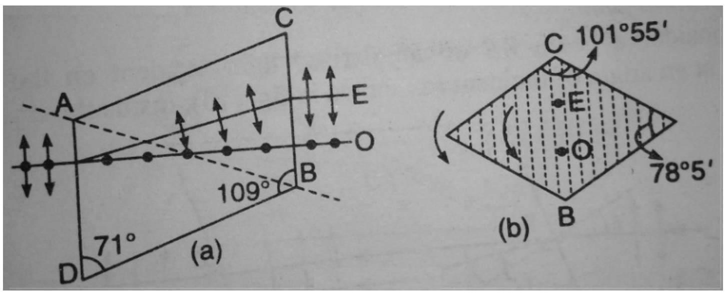

- Optic axis :- An imaginary line passing through any one of the blunt corners and making equal angles with the three faces which meet at this corner is called "optic axis".
- Optic axis is the direction but not a perticular line. So, all lines parallel this line are optic axes. These crystals are of two types.
- uniaxial crystals :- Crystals having only one optic axis are called uniaxial crystals. Ex: Calcite, Quartz, Tourmaline etc.
- Biaxial crystals :- Crystals having two optic axes are called biaxial crystals. Ex: Borax, Topaz, Mica etc.
- Principal section :- The plane that contains optic axis and perpendicular to the two opposite faces is called "principal section".
- The principal section cuts the crystal in to a parallelogram having angles $109^0$ and $71^0$.

## Double refraction

Definition :- when an unpolarised light ray incident on a crystal, it splits in to two refracted rays and travel with deferent velocities in different directions is called double refraction.

The crystals showing this property are called doubly refracting crystals.

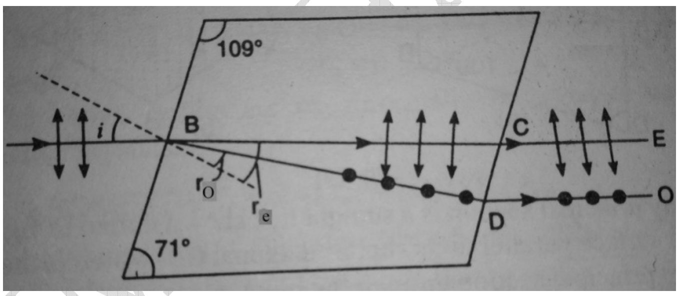
Ex :- Calcite, Quartz, Tourmaline, Mica etc.




**Explanation :-**

- ✓ An ink dot is made on a white paper and a calcite crystal is placed on the dot. If the dot is observed through the crystal it appears as two (Fig - b).

- ✓ In these two dots, one is due to ordinary light and the second one is due to extra-ordinary light.
- ✓ If the crystal is rotated in the anti-clock wise direction then one image (dot O) is stationary. This image is called ordinary image and this is formed due to ordinary light.
- ✓ The second image (dot E) rotates around the stationary image. This image is called extra-ordinary image and this is formed due to extra-ordinary light.
- ✓ Both ordinary and extra-ordinary rays are plane polarised lights.
- ✓ Ordinary light comes out as σ – components and extra-ordinary light comes out as π – components (Fig-a).
- ✓ So, by sending the unpolarised light through a calcite crystal, we can get two polarised lights i.e. ordinary and extra-ordinary lights.
- ✓ The ordinary light follows the laws of refraction and has the same velocity in all directions.
- ✓ But extra-ordinary light does not follow the laws of refraction and has different velocities in different directions.
- ✓ Let 'i' be the angle of incidence, $r_o$ and $r_e$ be the angles of refraction for ordinary and extra-ordinary rays.
- ✓ Let the velocity of light in air be C and $C_o$ & $C_e$ be the velocities of ordinary and extra-ordinary rays in the crystal.




Then, from Snell's law

Refractive index $\mu = \frac{\sin i}{\sin r} = \frac{C}{C'}$

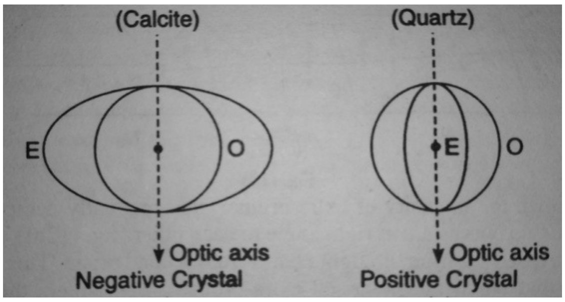
$C' = Velocity\ of\ light\ in\ the\ medium$

Refractive index of ordinary light

$$\mu_o = \frac{\sin i}{\sin r_o} = \frac{C}{C_o}$$

Refractive index of extra-ordinary light

$$\mu_e = \frac{\sin i}{\sin r_e} = \frac{C}{C_e}$$




If $r_e > r_o$ (or) $C_e > C_o$

Then $\mu_e < \mu_o$ (or) $\mu_e - \mu_o =$ Negative This crystal is called negative crystal.

If $r_e < r_o$ (or) $C_e < C_o$

Then $\mu_e > \mu_o$

(or) $\mu_e - \mu_o = Positive$

This crystal is called positive crystal.

| | |
|---|---|
| $r_e > r_o$ (or) $C_e > C_o$<br>$\mu_e - \mu_o =$ Negative<br>Negative crystal | $r_e < r_o$ (or) $C_e < C_o$<br>$\mu_e - \mu_o = Positive$<br>Positive crystal |

As per Huygen's theory, the spheres in the Figure are the wave fronts of ordinary light and the ellipsoids are the wave fronts of extra-ordinary light. So, the ordinary and extra-ordinary rays have the same velocity in the direction of optic axis.

## Polarisation due to scattering

The unpolarised light scatters when it is passed through a material medium. There are two reasons for this scattering. 1) Random reflection 2) Diffraction.

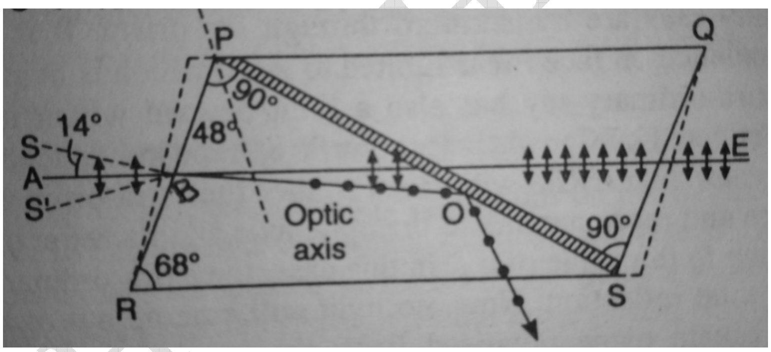
Diffraction will take place if the size of the molecule is small compared to the wave length of light.

If the scattered rays make $90^0$ with the incident beam then the scattered rays are plane polarised.

## Nicol Prism

- ❖ When unpolarised light is passed through Calcite crystal, it is divided in to two. 1) ordinary ray and 2) extra-ordinary ray. These two are plane polarised lights and come out from the opposite face of the crystal.
- ❖ The calcite crystal is modified to bring out only one polarised light from the opposite face of the crystal is called "Nicol prism".

**Construction** :-




- ➢ Nicol prism is also calcite crystal. But the ratio of its length to breadth is 3 : 1
- ➢ The faces PR & QS, opposite to its length are grounded such that its principal section is a parallelogram having angles $68^o$, $112^o$. This is done to increase the field view.
- ➢ This crystal is cut in two parts in a plane perpendicular to the principal section and passing through the two corners P & S and making $90^0$ with the faces PR & QS.
- ➢ The two cut faces are grounded and polished as optically plane surfaces. These two surfaces are cemented together with Canada balsam with out air bubbles in it.
- ➢ Canada balsam is a transparent material. Its refractive index $\mu = 1.55$. This value is in between the refractive index ordinary light $\mu_o = 1.658$ and refractive index of extra-ordinary light $\mu_e = 1.486$.

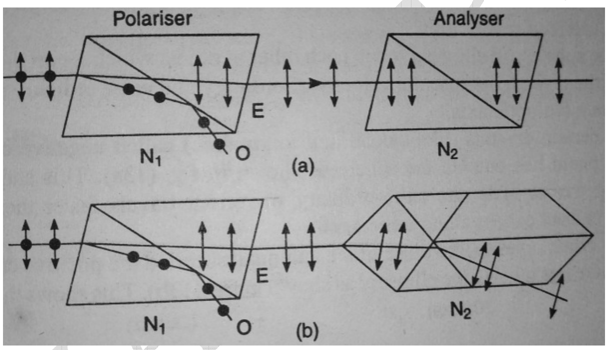
Working :-

- ✪ The unpolarised light incident on the surface PR is divided into ordinary and extra-ordinary rays and reach the Canada balsam layer. This layer acts as rarer medium to the ordinary ray and denser medium to the extra-ordinary ray.
- ✪ If the ordinary ray incident on the Canada balsam layer with more than the critical angle, then the ordinary ray got total internal reflection and reaches the RS surface and is absorbed by the lamp black at this surface.
- ✪ But the extra-ordinary ray refracted and emerges out from the QS surface. Like this the ordinary light can be eliminated and we can get extra-ordinary (polarised ) light.
- ✪ The ordinary light ray incident on Canada balsam at an angle $69^o$. This value is more than the critical angle. So, the ray takes total internal reflection.

**Limitations of Nicol prism** :-

- AB is the incident ray, if the angle of this ray with BR decreases as $\angle S'BR$ then the angle of incidence of ordinary ray at Canada balsam layer is less than the critical angle. So, ordinary ray did not get total internal reflection. So, both the ordinary and extra-ordinary rays refracted and emerge out from the QS surface.
- If the angle of incident ray with BR is increased as $\angle SBR$ then both the ordinary and extra-ordinary rays incident on Canada balsam layer with angle more than that of critical angle. So, both the rays got total internal reflection at Canada balsam layer and are absorbed by the lamp black at the RS surface. No light will come out from QS surface.

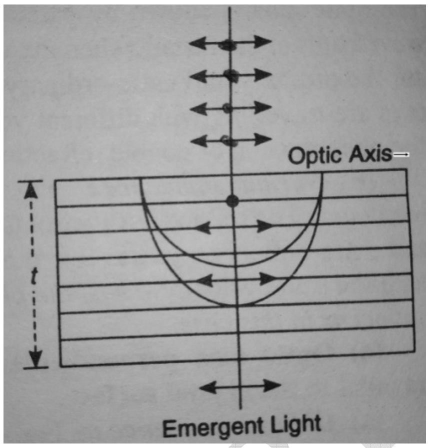
## Nicol prism as polarizer and analyser




- Take two Nicol prisms $N_1$, $N_2$ on a common axis such that their principal sections are confined to single plane.
- Nicol $N_1$ produces plane polarised light. So, is called polariser. Nicol $N_2$ analyse the polarised light. So, it is called analyser.
- Unpolarised light incident on $N_1$ and divided in to two polarised lights i.e. ordinary and extra-ordinary.
- The vibrations of ordinary light waves are perpendicular to the plane of the principal section of $N_1$ and leaves $N_1$ through the lower surface.
- But the vibrations of Extra-ordinary light waves are in the plane of the principal section of $N_1$ and leaves $N_1$ through the opposite surface. These vibrations of extra-ordinary rays are also in the plane of principal section of $N_2$. So, extra-ordinary rays emerge out through $N_2$ also with out any change. So, some light is present after $N_2$.
- If the Nicol $N_2$ is rotated through $90^0$ about the common axis, then the vibrations of extra-ordinary rays coming from $N_1$ are perpendicular the plane of principal section of $N_2$. So, this extra-ordinary rays become ordinary rays to $N_2$ and leaves $N_2$ through the lower surface. No light comes out from the opposite side of $N_2$ and is dark.
- So, it is clear that the light coming from $N_1$ is plane polarised light.

## Quarter wave plate

- ⊗ Doubly refracting crystal like calcite is taken such that its optic axis lies in its refracting or incident surface.
- ⊗ If the unpolarised light incident normally on the refracting surface of the crystal then this light is also normal to the optic axis.
- ⊗ The unpolarised light splits in to ordinary and extra-ordinary rays when it enters the crystal and travel in the same direction with different velocities as these rays are normal to the optic axis.




- ⊗ As calcite is negative crystal extra-ordinary ray has more velocity than ordinary ray.
- ⊗ So, the ordinary ray lags behind the extra-ordinary ray and the path difference (Δ) between them increases as they travel in the crystal.

  Let the thickness of the crystal be 't' and the refractive indices of extra-ordinary and ordinary rays be $\mu_e$ & $\mu_o$ respectively.

Then

Distance travelled by ordinary light = $\mu_o t$

Distance travelled by extra-ordinary light = $\mu_e t$

Path difference between them $\Delta = \mu_o t - \mu_e t$ (or) $\Delta = (\mu_o - \mu_e)\, t$

If this path difference is equal to $\frac{\lambda}{4}$, then that crystal is called "quarter wave plate".

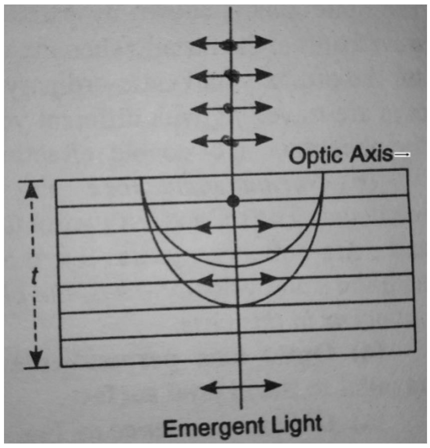
$$\therefore\ \Delta = (\mu_o - \mu_e)t = \frac{\lambda}{4}$$

$$\boxed{t = \frac{\lambda}{4\,(\mu_o - \mu_e)}}$$

**Definition** :- If the crystal splits the unpolarised light into ordinary and extra-ordinary rays and produces a path difference $\frac{\lambda}{4}$ between them, then that crystal is called "quarter wave plate".

If the colour of the light changes then its $\lambda$ also changes. So, different colours should have quarter wave plates of different thicknesses.

## Half wave plate




- ⊗ Doubly refracting crystal like calcite is taken such that its optic axis lies in its refracting or incident surface.
- ⊗ If the unpolarised light incident normally on the refracting surface of the crystal then this light is also normal to the optic axis.
- ⊗ The unpolarised light splits in to ordinary and extra-ordinary rays when it enters the crystal and travel in the same direction with different velocities as these rays are normal to the optic axis.
- ⊗ As calcite is negative crystal extra-ordinary ray has more velocity than ordinary ray.
- ⊗ So, the ordinary ray lags behind the extra-ordinary ray and the path difference (Δ) between them increases as they travel in the crystal.

  Let the thickness of the crystal be 't' and the refractive indices of extra-ordinary and ordinary rays be $\mu_e$ & $\mu_o$ respectively.

Then

Distance travelled by ordinary light = $\mu_o t$

Distance travelled by extra-ordinary light $= \mu_e t$

Path difference between them $\Delta = \mu_o t - \mu_e t$ (or) $\Delta = (\mu_o - \mu_e)\, t$

If this path difference is equal to $\frac{\lambda}{2}$, then that crystal is called "half wave plate".

$$\therefore\ \Delta = (\mu_o - \mu_e)t = \frac{\lambda}{2}$$

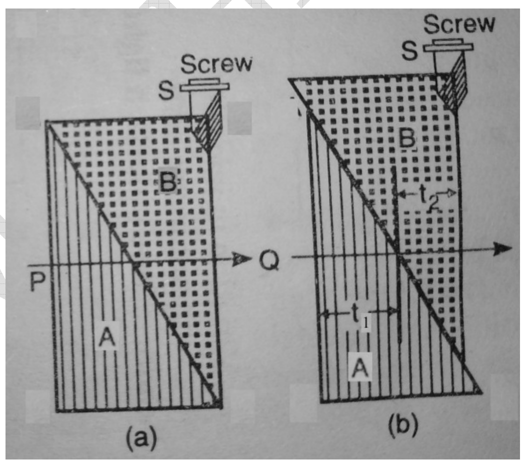
$$\boxed{t = \frac{\lambda}{2\,(\mu_o - \mu_e)}}$$

**Definition** :- If the crystal splits the unpolarised light into ordinary and extra-ordinary rays and produces a path difference $\frac{\lambda}{2}$ between them, then that crystal is called "half wave plate".

If the colour of the light changes then its $\lambda$ also changes. So, different colours should have half wave plates of different thicknesses.

## Babinet's compensator

- ❖ The thicknesses of quarter wave plate and half wave plate should be changed if the wave length or colour of light changes. Babinet's compensator is constructed to eliminate this draw back.
- ❖ So, Babinet's compensator can be used for any wave length of light. More over, any value of phase difference can be generated between the ordinary and extra-ordinary rays.




**Construction** :-

- ✪ Babinet's compensator consists of two wedge shaped quartz crystals A & B. A screw S, is attached to the crystal B such that the two wedge surfaces slide on one another.
- ✪ The optic axis of the crystal A is parallel to its 1st surface and lies in the plane of the paper. The optic axis of the crystal B is also parallel to its 2nd surface but perpendicular to the plane of the paper and also perpendicular to the optic axis of crystal A.
- ✪ The incident light ray PQ is monochromatic and unpolarised. This ray is incidenting normally to both the optic axes of the crystals A and B.
- ✪ So, this ray is divided into ordinary and extra-ordinary rays and both will travel in the same direction. Since Quartz is positive crystal, the ordinary ray has more velocity than the extra-ordinary ray.

  Let the refractive indices of extra-ordinary and ordinary rays be $\mu_e$ & $\mu_o$ respectively.

  If the two rays travel $t_1$ distance in crystal A,

  then the path difference between them $\Delta_1 = (\mu_e - \mu_o)t_1$
- ✪ If these two rays enter into the 2nd crystal B, then the ordinary ray becomes extra-ordinary ray and extra-ordinary ray becomes ordinary ray.
- ✪ The reason for this is that the two optic axes in A and B are mutually perpendicular.

Note :- If the vibrations of light wave is in the plane of the optic axis then the light is extra-ordinary light and if the vibrations of light wave is perpendicular to the plane of the optic axis then the light is ordinary light.

If the two rays travel $t_2$ distance in crystal B,
then the path difference between them $\Delta_2 = -(\mu_e - \mu_o)t_2$

Then the total path difference $\Delta = \Delta_1 + \Delta_2 = (\mu_e - \mu_o)t_1 - (\mu_e - \mu_o)t_2$

(or) $\boxed{\Delta = (\mu_e - \mu_o)(t_1 - t_2)}$

Similarly the total phase difference $\delta = \frac{2\pi}{\lambda}.\Delta$

$$\therefore \delta = \frac{2\pi}{\lambda}.(\mu_e - \mu_o)(t_1 - t_2)$$

The crystal B can be raised up or lowered down by turning the screw S. Then $t_2$ value and $(t_1 - t_2)$ value change. So, the Babinet's compensator can be used for any wave length of light. More over, any path difference can be produced between the ordinary and extra-ordinary rays by using this.

## Optical activity

- Two Nicols, N1 and $N_2$, those act as polarizer and analyser are placed in crossed position on a common axis. If an unpolarised light placed before them, is viewed through the analyser, the field of view is completely dark.
- If a quartz plate whose optic axis is in the plane of the incident surface, is placed between the two Nicols, the field of view is not dark. If the analyser is rotated through some angle once again the field of view is dark.
- That means the light emerging out from the quartz crystal is also plane polarized but the plane of polarization is rotated through some angle.

**Definition**:- The property of rotating the plane of polarization of plane polarized light about the direction of propagation by a material is called as optical activity or optical rotation.

These materials are called optically active materials. The angle through which the plane of polarization rotated is called the angle of rotation.

There are two types of optically active substances.

1. Dextro- rotatory substances:- These materials rotate the plane of polarization in the clock-wise direction and are also called as right handed rotatory or positive rotatory substances.
2. Laevo- rotatory substances:- These materials rotate the plane of polarization in the anti-clock-wise direction and are also called as left handed rotatory or negative rotatory substances.

If light passes through different optically active substances, then the total angle of rotation is equal to the algebraic sum of the individual angles of rotation. $\theta = \theta_1 + \theta_2 + \theta_3 - - -$

The angle of rotation ($\theta$) is directly proportional to length (l) of travel in the substance.

$$\theta \propto l$$

If the substance is liquid or vapour then $\theta$ is directly proportional to the concentration (C) of the liquid or vapour.

$$\theta \propto C$$

θ is also inversely proportional to the square of the wave length ( λ) of the light used.

$$\theta \propto \frac{1}{\lambda^2}$$

θ also increases with the temperature of the substance.

For a particular wave length of light and particular temperature of the substance,

$$\theta \propto l\,C \quad \text{or} \quad \theta = S\,l\,C$$

Here S is proportionality constant and is called specific rotation.

$$\boxed{S = \frac{\theta}{l\,C}}$$

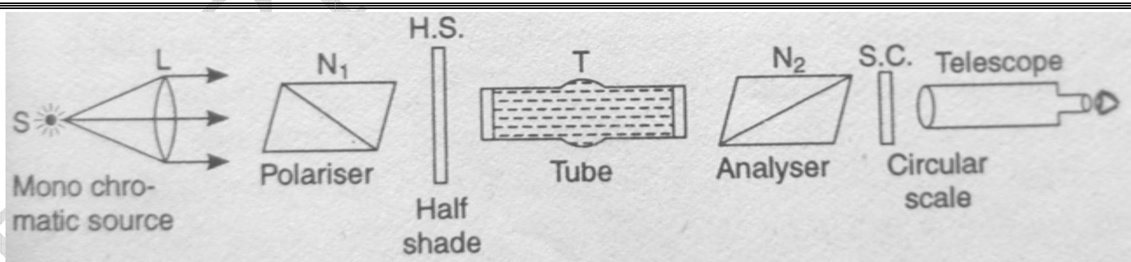
**Specific rotation** :- The specific rotation of the substance at a particular temperature for a given wave length of light used is the angle of rotation of the plane of polarization per unit concentration of the liquid and per unit length of travel in deci metres.

Or $S = \frac{10\,\theta}{l\,C}$ Here l is in cm.

## Laurent's half shade polarimeter

Polarimeters are used to measure the angle of rotation and from that the specific rotation of the substance can be calculated. If the scale is calibrated to give concentration directly, then it is called saccharimeter.

**Construction**:-

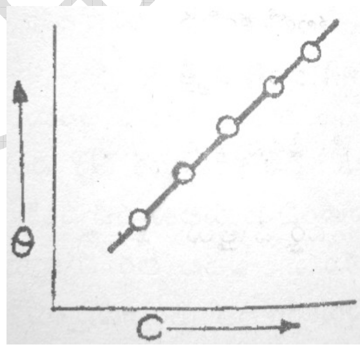



- This polarimeter has a source of monochromatic light. This gives a diverging beam of light. This diverging beam falls on a convex lens and is converted in to a parallel beam of light.
- After that two Nicols $N_1$ & $N_2$ are arranged coaxially and they act as polariser and analyser.
- In between $N_1$ & $N_2$ a half shade device and a glass tube (T) having more diameter at the centre are placed. The tube T is filled with the optically active liquid.
- $N_2$ can be rotated about the common axis. The angle of rotation can be measured with a circular scale and vernier.

- The light from the analyser $N_2$ is viewed through a telescope.

**Working** :-

- First the tube T of length 'l' is filled with pure water and it is placed in its position.
- The telescope is focused on the half shade and now the two halves in the half shade plate are not equally bright. Then the analyser $N_2$ is rotated until the two halves become equally bright and the reading $R_0$ on the circular scale is taken.
- Now the water in the tube is replaced with the optically active solution of known concentration ($C_1$).
- Once again the telescope is focused and $N_2$ is rotated until the two halves in the half shade plate become equally bright and the reading $R_1$ on the circular scale is taken.
- The angle of rotation for the concentration $C_1$ is $\theta_1 = (R_1 \sim R_0)$.
- The experiment is repeated with different known concentrations of solution $C_2$, $C_3$ …. and the readings on circular scale at equal brightness are noted as $R_2$, $R_3$ ….
- The angles of rotation are $\theta_2 = (R_2 \sim R_0)$, $\theta_3 = (R_3 \sim R_0)$…. etc.
- A graph is drawn by taking concentration C on X-axis and angle of rotation θ on Y-axis and it gives a straight line passing through the origin. Select one concentration C in the graph and take its corresponding angle of rotation θ.
- Substitute these values C, θ and length of the tube l in the below equation and specific rotation of the solution can be found.

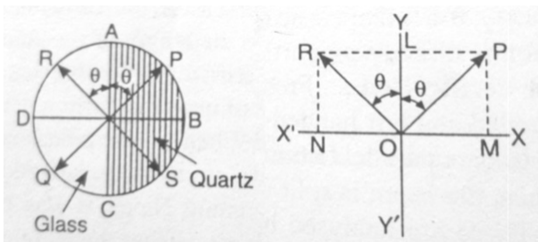



$$S = \frac{10\,\theta}{l\,C}$$

**Laurent's half shade device** :- The measurement of optical rotation is not accurate if only polarizer and analyzer are used in polarimeter. In order to avoid this error, half shade device is used.

Construction :-

- ✓ Laurent half shade device consists of two parts. 1) Semi circular half wave plate ABC, made with quartz & its optic axis is parallel to the straight edge 2) Semicircular glass plate ADC.
- ✓ These two semi circular plates are cemented to form circular plate. These two parts absorb the same amount of light.




Working :-

- ✓ Let the plane of vibration of plane polarized light from $N_1$ be along PQ and it makes angle θ with edge AC (Optic axis).

- ✓ This is represented by OP in the fig. This 'OP' is the direction of vibration in glass plate too.
- ✓ When this OP enters in to the quartz plate, it is divided in to two components , one extra-ordinary along OL, the other ordinary along OM.
- ✓ But the ordinary ray moves with more velocity than extra-ordinary ray in quartz, as quartz is a positive crystal.
- ✓ When the rays emerged out from the quartz the O-ray travels an additional path of $\lambda/2$ or the phase of $\pi$ than that of E-ray as quartz acts as half wave plate.
- ✓ So, the ordinary ray component is reversed as ON. But there is no change in the extra-ordinary component i.e. along 'OL'. So the resultant of these two is 'OR' emerged out from the quartz.

Cases :-

1. If the principal section of the analyser $N_2$ is parallel to OP, then the light from the glass portion is more bright than the light from the quartz portion.
2. If the principal section of the analyser $N_2$ is parallel to OR, then the light from the quartz portion is more bright than the light from the glass portion.
3. If the principal section of the analyser $N_2$ is parallel to OL, then the two portions are equally bright.

- ➢ So, the half-shade device divides the field of view into halves.

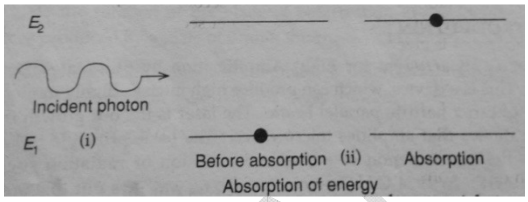
## Chapter – VII

# LASERS

The expansion of LASER is "Light Amplification by Stimulated Emission of Radiation". In this the light is amplified by stimulated emission.

**Absorption of radiation** :-

- Consider two energy states 1and 2 of an atom having energies $E_1$ and $E_2$. Before absorbing the energy the electron or the atom is said to be in the ground state.




- If an amount of energy, $E_2$-$E_1$ = hν , is given to the atom then the electron absorbs that energy and goes from state 1 to state 2. This state 2 is called excited state.
- This absorption of energy is called stimulated absorption or induced absorption.
- Here h = Planck's constant, ν = frequency of the photon(Packet of energy) (or) hν = Energy of photon.
- The electron can be sent from ground state 1 to excited state 2 by two methods. 1) colliding the atom with some other energetic particle. Here the kinetic energy of the colliding particle is transferred to the electron. So, the electron goes to excited state. 2) The electron absorbs the photon that incident on it and goes to excited state.
- The electron stay in the excited state for $10^{-8}$ sec or 10 ns. This time is called the life time of the excited state.

No. of atoms excited by stimulated absorption per unit time is

$N_{ab} \propto N_1$ — $N_1$ = No. of atoms in ground state

$\propto \rho(\nu)$ — $\rho(\nu)$ = Energy density of radiation

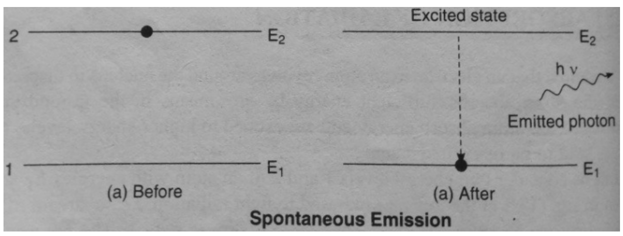
(or) $N_{ab} = B_{12} N_1 \rho(\nu)$ — $B_{12}$ = Proportionality constant

**Emission of radiation** :-

- When the electron jumps from excited state 2 to the ground state 1,then the atom emits some energy, $E_2 - E_1$ = hν ( Photon). This is called emission of radiation.
- The emission of radiation is of two types 1) Spontaneous emission 2) Stimulated emission

**Spontaneous emission** :-

- The electron can not stay in the excited state 2 more than its life time ($10^{-8}$s). So, it takes transition from excited state 2 to the ground state 1 on its own by emitting energy $E_2 - E_1$ = hν This is called Spontaneous emission.

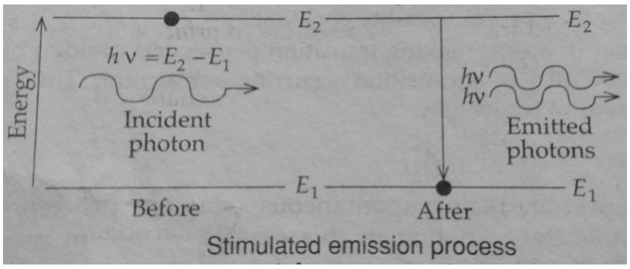



No. of atoms went from excited state to ground state by spontaneous emission per unit time is

$N_{sp} \propto N_2$ — $N_2$ = No. of atoms in excited state 2

(or) $N_{sp} = A_{21} N_2$ — $A_{21}$ = Proportionality constant

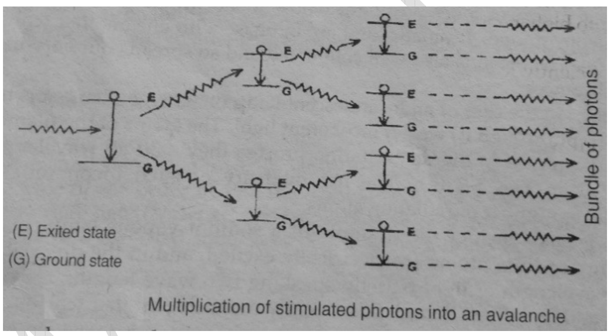
**<u>Stimulated emission</u>** :-



Stimulated emission process

- ∗ The electron in the excited state need not wait for spontaneous emission. If the excited electron is hit by a photon of energy, $E_2 - E_1 = h\nu$ then the electron in the excited state comes to ground state by emitting some energy $E_2 - E_1 = h\nu$. This is called stimulated emission or induced emission.

- ∗ Here two photons are of energy $E_2 - E_1 = h\nu$ are released. One photon is due to the transition of the electron and the second photon is the incident photon.



Multiplication of stimulated photons into an avalanche

- ∗ These two photons trigger two more excited electrons giving 4 photons. Like this, due to stimulated emission the no. of photons released increase in geometric progression.
- ∗ If all these photons are monochromatic, coherent and have same direction, then it is LASER.

No. of atoms went from excited state to ground state by stimulated emission per unit time is

$N_{st} \propto N_2$      $N_2$ = No. of atoms in excited state

$\propto \rho(\nu)$      $\rho(\nu)$ = Energy density of radiation

(or)    $N_{st} = B_{21} N_2 \rho(\nu)$      $B_{21}$ = Proportionality constant

Under thermal equilibrium condition

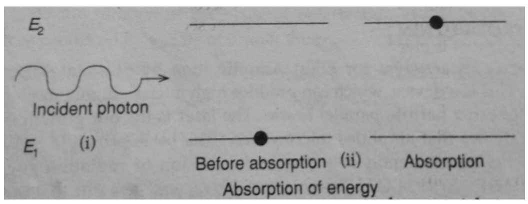
No. of upward transitions = No. of down ward transitions

(or)    $N_{ab} = N_{sp} + N_{st}$

**<u>Einstein coefficients</u>** :-

1. <u>Induced absorption</u>

Let $E_1$& $E_2$ be the energies of electron when it is in states 1 & 2. If an amount of energy, $E_2$-$E_1$ = hν , is given to the atom then the electron absorbs that energy and goes from state 1 to state 2. This state 2 is called <u>excited state</u>.



Absorption of energy

This absorption of energy is called stimulated absorption or induced absorption.

No. of atoms excited by stimulated absorption per unit time is

$N_{ab} \propto N_1$      $N_1$ = No. of atoms in ground state

$\propto \rho(\nu)$      $\rho(\nu)$ = Energy density of radiation

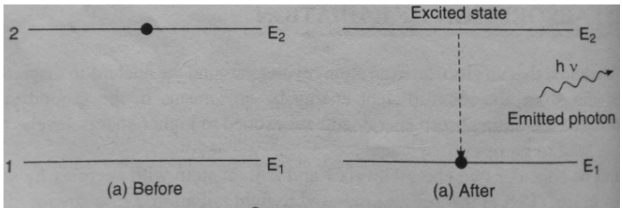
(or)    $N_{ab} = B_{12} N_1 \rho(\nu)$      $B_{12}$ = Proportionality constant

This proportionality constant is called <u>Einstein coefficient of induced absorption</u>.

The suffix 12 means the transition is from state 1 to state 2.

2. Spontaneous emission

The electron can not stay in the excited state 2 more than its life time ($10^{-8}$ s). So, it takes transition from excited state 2 to the ground state 1 on its own by emitting energy $E_2 - E_1 = h\nu$ This called Spontaneous emission.



Spontaneous Emission

No. of atoms went from excited state to ground state by spontaneous emission per unit time is

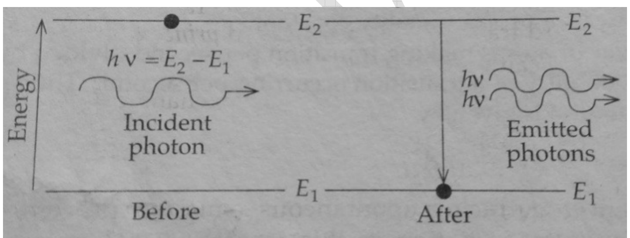
$N_{sp} \alpha N_2$ $\qquad$ $N_2$ = No. of atoms in excited state

(or) $N_{sp} = A_{21} N_2$ $\qquad$ $A_{21}$ = Proportionality constant

This proportionality constant is called Einstein coefficient of spontaneous emission.
The suffix 21 means the transition is from state 2 to state 1.

3. Stimulated emission

The electron in the excited state need not wait for spontaneous emission. If the excited electron is hit by a photon of energy, $E_2 - E_1 = h\nu$ then the electron in the excited state goes to ground state by emitting some energy $E_2 - E_1 = h\nu$. This is called stimulated emission or induced emission.



Stimulated emission process

No. of atoms went from excited state to ground state by stimulated emission per unit time is

$N_{st} \alpha N_2$ $\qquad$ $N_2$ = No. of atoms in excited state

$\alpha\, \rho(\nu)$ $\qquad$ $\rho(\nu)$ = Energy density of radiation

(or) $N_{st} = B_{21} N_2 \rho(\nu)$ $\qquad$ $B_{21}$ = Proportionality constant

This proportionality constant is called Einstein coefficient of stimulated emission.
The suffix 21 means the transition is from state 2 to state 1.

Relation among Einstein coefficients

Under thermal equilibrium condition

No. of upward transitions = No. of down ward transitions

(or) $N_{ab} = N_{sp} + N_{st}$

$$B_{12} N_1 \rho(\nu) = A_{21} N_2 + B_{21} N_2 \rho(\nu)$$

$$\rho(\nu) [B_{12}N_1 - B_{21} N_2] = A_{21} N_2$$

$$\rho(\nu) = \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]}$$

Dividing the numerator and denominator by $B_{21}N_2$

Then $$\rho(\nu) = \frac{\frac{A_{21}N_2}{B_{21}N_2}}{\left[\frac{B_{12}N_1}{B_{21}N_2} - 1\right]} = \frac{\frac{A_{21}}{B_{21}}}{\left[\frac{B_{12}}{B_{21}}\frac{N_1}{N_2} - 1\right]} \longrightarrow (1)$$

At thermal equilibrium, according to Boltzmann distribution law, the no. of atoms in the state 1 is given by $N_1 = N_0\, e^{-\frac{E_1}{kT}}$

Here $E_1$ = Energy of state 1

k = Boltzmann constant

T = absolute temperature

Similarly, the no. of atoms in state 2 is given by $N_2 = N_0\, e^{-\frac{E_2}{kT}}$

Then $\frac{N_1}{N_2} = \frac{N_0\, e^{-\frac{E_1}{kT}}}{N_0\, e^{-\frac{E_2}{kT}}} = e^{\frac{E_2 - E_1}{kT}}$

But $E_2 - E_1 = h\nu$

$$\therefore \frac{N_1}{N_2} = e^{\frac{h\nu}{kT}} \longrightarrow (2)$$

Substituting eqn. (2) in eqn. (1)

$$\rho(\nu) = \frac{\frac{A_{21}}{B_{21}}}{\left[\frac{B_{12}}{B_{21}}\; e^{\frac{h\nu}{kT}} - 1\right]}$$

Einstein proved that the probabilities of stimulated absorption and stimulated emission are equal.

i.e. $B_{12} = B_{21}$

$$\therefore \rho(\nu) = \frac{\frac{A_{21}}{B_{21}}}{\left[e^{\frac{h\nu}{kT}} - 1\right]} \longrightarrow (3)$$

According to Planck's radiation law $\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left[e^{\frac{h\nu}{kT}} - 1\right]} \longrightarrow (4)$

C = velocity of light

Comparing the eqns. (3) & (4) $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$ (or) $\boxed{A_{21} \propto \nu^3}$

This equation says that the spontaneous emission increases with the increase of energy difference between the excited and ground states.

**Population inversion**

- ✓ In general the no. of atoms in the ground state $N_1$ are more than the no. of atoms in the excited state $N_2$.
- ✓ Definition : - Making of the no. of atoms in the excited state $N_2$ more than the no. of atoms in the ground state $N_1$ is called population inversion. Here ($N_2 > N_1$).
- ✓ The process of achieving population inversion is called pumping. There are different methods of pumping.
  1. Optical pumping (Ruby laser)
  2. Electric discharge (He -Ne laser)
  3. Direct conversion (Semi-conductor laser)
  4. Chemical reaction ($CO_2$ laser)

**Meta-stable state**

- ✪ In order to achieve population inversion the electron should stay more time in the excited state i.e. more than the life time of the excited state ($10^{-8}$s). This type of state is called "meta-stable state".
- ✪ The life time of meta-stable state is in the order of $10^{-3}$ s (or) 1 ms.
- ✪ Meta-stable state can be obtained by adding impurity to a crystal. The meta-stable state lies in the forbidden band of the host crystal.



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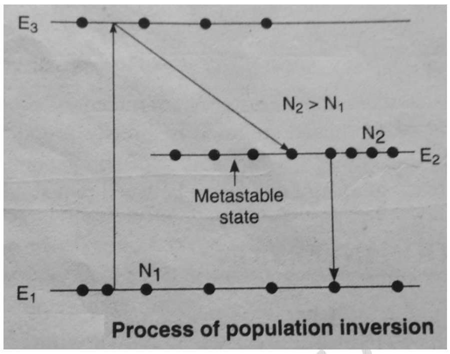


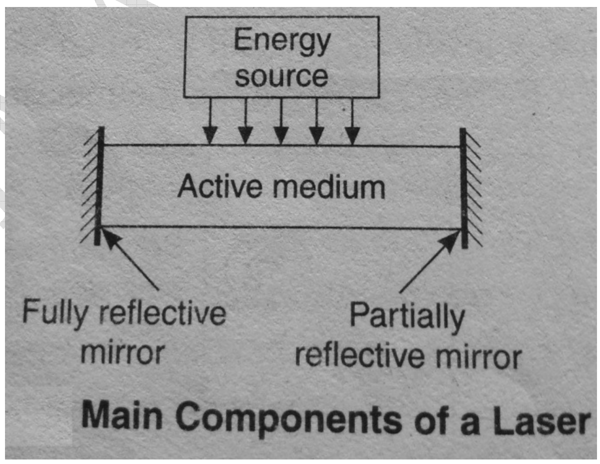

**Principle of laser** :-

Laser can be obtained in three steps

1. Pumping :- Supplying energy $E_3$-$E_1$ = hν to the electrons in the ground state, having energy $E_1$ and sending them to the excited state, having energy $E_3$, is called pumping. This is nothing but stimulated absorption.
2. Population inversion :- The electrons in the excited state 3 can not stay more time in that state. So, they come to the meta-stable state 2 by spontaneous emission. Electrons stay more time in meta-stable state. So, population inversion occurs between $E_1$ and $E_2$.
3. Stimulated emission :- one of the electrons in the meta-stable state $E_2$ transits to ground state $E_1$ by spontaneous emission by emitting a photon of energy $E_2$-$E_1$ = hν, this photon triggers the other electrons in state 2 for stimulated emission and give laser.

**Main components in laser apparatus**

Laser producing apparatus consists of three main components.




1. Active medium :- The medium used in which excited atoms and population inversion takes place is called active medium. This medium may be solid, liquid or gas.
2. Energy source :- The energy source required to excite the atoms from ground state to excited state.
3. Optical resonator :- This optical resonator consists of two mirrors facing each other. One is fully reflective and the other is semi transparent. This arrangement increases the intensity of the laser.

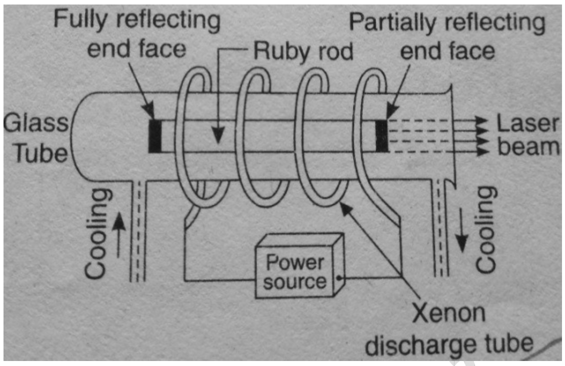
## Ruby laser

Ruby belongs to the family of gems. Ruby crystal consists of aluminium oxide ($Al_2O_3$) and in this 0.5% of aluminium atoms are replaced by chromium atoms ($Cr_2O_3$). Here $Cr^{3+}$ ions are active material. The colour of ruby crystal changes with the concentration of chromium.

Construction

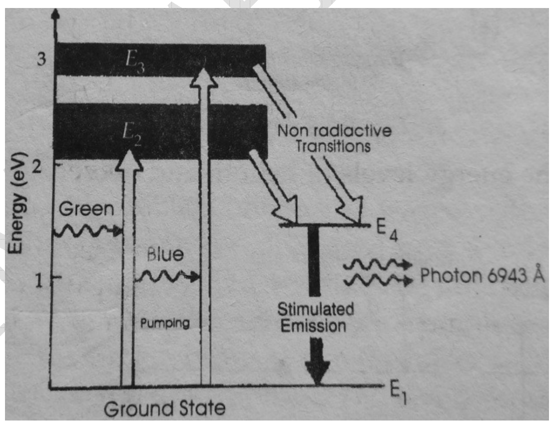
- ✪ Pink Ruby crystal is taken in the form of a cylindrical rod. Its length is 10 cm and diameter is 1 cm.
- ✪ The end faces of the rod are grounded and polished to a high degree such that the two faces are parallel. One face is fully silvered and that it completely reflects. The opposite face is silvered partially and it reflects partially.

- ✪ This rod is placed in coolant, like liquid $N_2$. This total arrangement is surrounded by a helical Xenon flash lamp connected to a power supply.
- ✪ The light from the Xenon lamp raise the electrons to excited state in Chromium ions.




Working

- ☞ The Chromium ions have three energy states, $E_1$ is ground state and $E_2$, $E_3$ excited states. The other state is meta-stable state $E_4$.
- ☞ The Xenon light gives very high intense white light. But the electrons in the ground state $E_1$ absorb only green and blue lights and transit to the excited states $E_3$, $E_4$ respectively.
- ☞ The life time of $E_2$, $E_3$ are very small, so the electrons in these states take radiationless transition to the meta-stable state $E_4$. This state has long life time.
- ☞ The population in $E_4$ exceeds the population of $E_1$ i.e. population inversion takes place between these two states.
- ☞ Then spontaneous emission takes place by the transition of some of the electrons from state $E_4$ to state $E_1$ by emitting photons of wave length 6943 $A^o$.
- ☞ These photons trigger the other excited chromium ions for stimulated emission. All these emitted photons move along the axis of the Ruby crystal rod and take multiple reflections between the two reflecting faces.




- ☞ If the photon beam is developed to a high intensity, then it comes out from the partially silvered face of the Ruby crystal as laser.

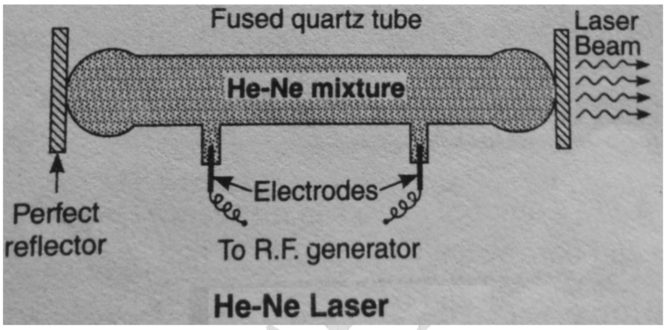
Drawbacks :-

1. This laser requires greater energy for excitation to get population inversion.
2. The Xenon flashes only for few milli seconds. So, laser beam is in pulses but not continuous.
3. Since, only the green component of the incident light is used, its efficiency is very less.
4. In addition crystalline imperfection also present in this.

**He – Ne laser**

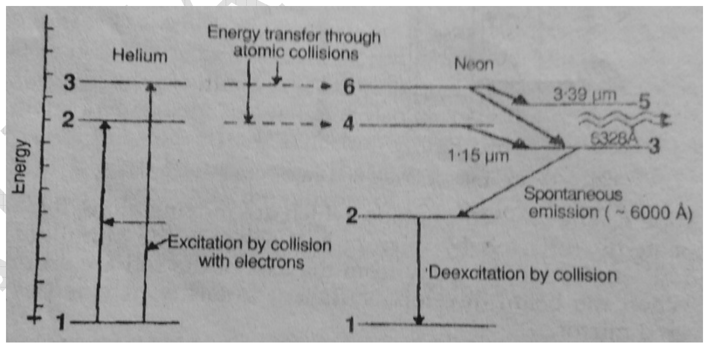
Construction

- This experiment consists of a discharge tube of 80cm length and 1.5 cm diameter made with fused quartz material.
- The tube is filled with the mixture of He & Ne gases in the ratio 10 : 1. A perfect reflector is placed at one end of the tube and a partial reflector is placed at the other end. These two reflectors are perfecty parallel.
- Two electrodes are inserted in to the gas and connected to a high frequency power supply for electric discharge.




Working

- When discharge is passed through the gas mixture, He atoms are excited to its $He_2$ and $He_3$ states from $He_1$ ground state. $He_2$ and $He_3$ states are meta-stable states. No allowed transitions from these two states.
- The excited He atoms collide inelastically with the Ne atoms and go to ground state by transfer of energy to Ne atoms.




- The Ne atoms go to $Ne_6$ and $Ne_4$ excited meta-stable states having energies nearly equal to $He_3$ and $He_2$ states respectively.
- The population inversion takes place between $Ne_6$, $Ne_4$ and $Ne_5$, $Ne_3$ as $Ne_6$, $Ne_4$ states are more populated and $Ne_5$, $Ne_3$ states are less populated.
- Here three laser transitions are possible.

$Ne_6 \rightarrow Ne_3$ gives laser beam of 6328 $A^0$.
$Ne_6 \rightarrow Ne_5$ gives laser beam of 33,900 $A^0$.
$Ne_4 \rightarrow Ne_3$ gives laser beam of 11,500 $A^0$.

- ✓ Laser beams of 33,900 $A^0$ and 11,500 $A^0$ are absorbed by the discharge tube and laser beam of 6328 $A^0$ only comes out.
- ✓ $Ne_5$, $Ne_3$ states take transitions to $Ne_2$ state by spontaneous emission. $Ne_2$ state is meta-stable state and it gives energy to discharge tube by collision and goes to ground state $Ne_1$.

Applications of He-Ne Lasers :-

1) He-Ne lasers are used in interferometers.
2) These are used in bar code scanners.
3) These are used in meteorology.

Applications of Lasers :-

1. Lasers are used in microwave communication because their band width is narrow. Laser radiation is not absorbed by water. So, lasers are also used for under water communication.
2. Lasers are used for drilling holes and cutting metals in industry.
3. Lasers are used to measure distances i.e. Light Detection And Ranging (LIDAR).
4. Pictures of clouds and wind movements can be obtained by using lasers and this data is used in weather forecasting.
5. Lasers are used in 3 dimensional photography i.e. holography.
6. Diode lasers are used to write and read the digital information on CDs and DVDs.
7. Lasers have many applications in medicine.
   i. Lasers are used for cataract operations and to weld retina with the eye ball.
   ii. Lasers are used for bloodless surgery and also in cancer diagnosis and treatment.
   iii. Lasers are used to destroy the stones in kidneys.

# Chapter – VIII

## Fibre Optics

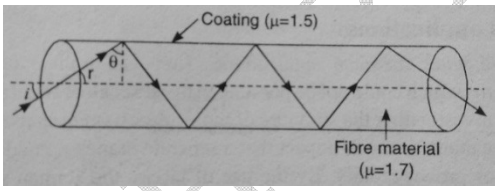
Fibre optics is the fastest communication system in which transparent dielectric medium acts as transmission medium and light acts as signal carrier.

Optical fibre :- Optical fibre is a thin strand (hair thin cylindrical strand) made of glass or transparent plastic. This acts as the medium through which the information is transmitted.

- ❖ Optical fibre do the similar work as the copper cable do in telephone conversation.

**Principle of optical fibre** :-

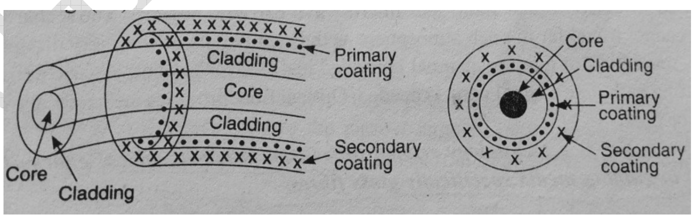
- ✪ Optical fibre is a transparent material (core) that acts as wave guide for light.
- ✪ The optical fibre is coated with a layer (cladding) of lower refractive index.
- ✪ Optical fibre works basing on the principle of total internal reflection.




- ✪ A light ray incident on the optical fibre and makes an angle 'i' with its axis. This ray incident on the boundary of the fibre with an angle of incidence "θ" more than that of critical angle. So, total internal reflection take place at the boundary.
- ✪ This type of total internal reflection is repeated and the light ray reaches to the other end of the fibre.

**Structure of optical fibre**

- ➢ An optical fibre consists of an inner cylinder that is made of glass or transparent dielectric material, called "core". Light travels through this core. The diameter of the core varies from 5μm to 100μm.




- ➢ There is another cylindrical shell around the core called cladding. This cladding has the lower refractive index than that of core.
- ➢ Cladding helps the light to take total internal reflections in the core. The diameter of the cladding is 125μm.
- ➢ To provide strength and protection for the fibre, two plastic coatings are given above the cladding and these are called primary and secondary coating.

**Types of optical fibres**

Optical fibres are classified in three different ways.

- I. Basing on the materials used
- II. Basing on the modes of propagation.
- III. Basing on the refractive index of the core.

I. Classification basing on the materials used

1. Glass fibres :-
   - ☞ In these fibres the core and cladding both are made with glass. The glass used should be highly pure and highly transparent. Ex:- Silicon dioxide, Fused quartz.
   - ☞ Impurities should be added to the pure glass to change the refractive index.

☞ Germanium and Phosphorous impurities increase the refractive index of glass (core) and Boron and Fluorine impurities decrease the refractive index of glass(cladding).

2. Plastic clad – silica fibres :-

✓ In these fibres the core is glass and the cladding is plastic. The performance of these fibres not good.

3. Plastic fibres :-

✪ In these fibres both the core and the cladding are made with plastic material.

✪ Their cost is low. It is very easy to use them. These can be used where less band width is sufficient. They have electromagnetic immunity. The transmission losses are also very less.

II. Classification basing on the modes of propagation.

⊕ Mode is a path that a light ray travels in an optical fibre.

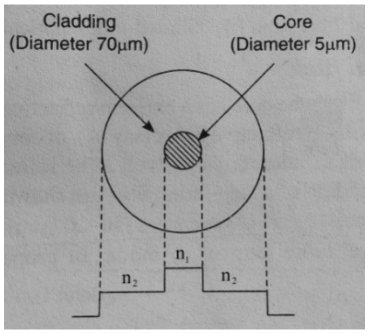
⊕ $No. of\ modes\ that\ a\ fibre\ supports\ \propto \frac{d}{\lambda}$

d = diameter of the core λ = wave length of the light used.

1. Single mode fibres (SMF) (or) Monomode fibres:-

⊗ Single mode fibre has smaller core diameter 5μm. The cladding diameter ranges from 70μm to 125μm.

⊗ The refractive index of core ($n_1$) is greater than that of the cladding ($n_2$).




⊗ In this fibre only one mode can be propagated through the core.

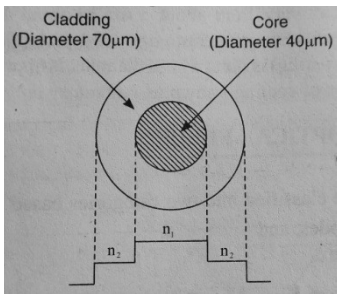
⊗ Transmission capacity is inversely proportional to dispersion. Dispersion causes noise.

⊗ But there is no dispersion in this fibre. So, no noise is created in this fibre. So, this fibre is the only choice for long distance transmission.

⊗ The light is passed in to the fibre through diode lasers. The fabrication of this fibre is difficult. So, these are more expensive.

2. Multimode fibres (MMF) :-

➢ Multimode fibre has larger core diameter 40 μm to 50μm than that in single mode. The cladding diameter ranges from 70μm to 125μm.




➢ The difference in the refractive indices between core and cladding ($n_1$ - $n_2$) is relatively large in this multimode fibre than that in single mode fibre.

➢ This fibre allows large no of modes as its core diameter is large.

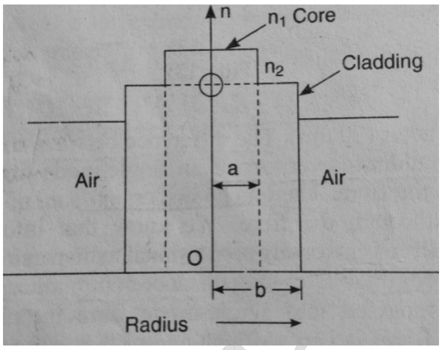
➢ Here dispersion takes place. This dispersion creates noise. So, this fibre can not be used for long distance transmission.

➢ The light is passed in to the fibre through LED source. The fabrication of this fibre is not difficult. So, these are less expensive.

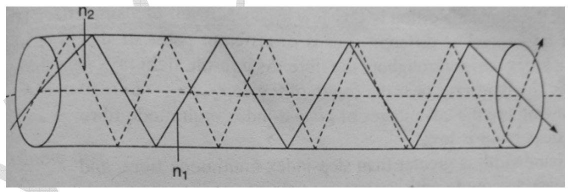
III. <u>Classification basing on the refractive index of the core.</u>

1. <u>Step index optical fibre</u>:-

- In step index optical fibre the core has higher constant refractive index ($n_1$) through out the core. The cladding has relatively lower constant refractive index ($n_2$) through out the cladding.



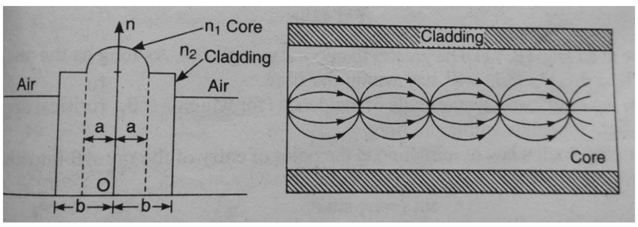

- As the refractive <u>index</u> increases one <u>step</u>, from cladding to core, this fibre is called "<u>step index</u>" optical fibre. The variation in the refractive indices of core and cladding are shown in the figure.
- This fibre has highest core diameter i.e. 50μm to 100 μm. So, it can propagate more no. of modes and it is also called as "multi-mode step index optical fibre".
- The diameter of the cladding is from 110 μm to 125 μm.
- For single mode, the core diameter is 10 μm.
- The light rays propagate in straight line paths between two successive reflections at core - cladding boundary.
- Two light rays entered into the core at the same time with different angles travel different distances and leave the core at different times in different directions.




- Even the rays enter the core with different angles, they have same velocities.
- But the ray entered into the core with high angle of incidence with the axis of the core, takes more no. of reflections and travel more distance. This ray reaches the other end with delay. So, the distortion is high.
- Advantages
  i. Easy and cheaper to manufacture
  ii. Longer life time
- Disadvantages
  i. Lower band width
  ii. High dispersion and noise
  iii. Can not be used for long distance transmission

2. <u>Graded index optical fibre</u> :-

- If the refractive index of the core ($n_1$) is non-uniform and decreases gradually from its centre to the core-cladding boundary, the fibre is called graded index optical fibre.
- The cladding has lower constant refractive index ($n_2$)
- The variation in the refractive indices of core and cladding are shown in the left figure.



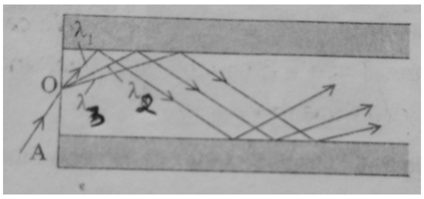

- For multimode transmission, the core and cladding diameters are 50 μm and 100μm respectively.

- The light rays travel in spiral paths or helical paths and they do not cross the axis of the core at any time.
- The light rays continuously bend in the core as the core refractive index gradually decreases. The motion is periodical. This was shown in right figure
- Even though the rays enter the core with different angles and travel in different paths, they have same time period.
- All the rays enter the core at one time will leave the core at one time even though they travel different distances. So, the distortion is low.
- The light rays go near to the boundary have more speed than the rays travel near the axis.
- Advantages
  i. Dispersion and noise are low
  ii. Band width is high
  iii. Can be used for long distance transmission
- Disadvantages
  i. Very expensive
  ii. Very difficult to manufacture

**Rays and modes in optical fibre**

Step index fibre

- In step index optical fibre the core has higher constant refractive index ($n_1$) through out the core. The cladding has relatively lower constant refractive index ($n_2$) through out the cladding.
- Consider an incident ray 'AO' consists of three different wave lengths $\lambda_1$, $\lambda_2$ & $\lambda_3$. When the incident ray AO enters in to the core it is dispersed in to three rays.
- The ray with wave length $\lambda_1$ incident on the core-cladding boundary with critical angle 'C'.
- The other two rays with wave lengths $\lambda_2$ & $\lambda_3$ incident on the core-cladding boundary with more than critical angle 'C'.

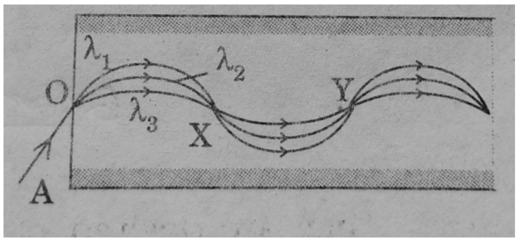



- They have same velocity through out the core as the core refractive index is constant and rays propagate in straight line paths between any two successive reflections at core - cladding boundary.
- The ray incident on core-cladding boundary with large angle of incidence will take less no. of reflections and travel less distance in the core and reaches the other end earlier.
- But the rays incident on core-cladding boundary with less angle of incidence take more no. of reflections and travel more distance. They ray reach the other end with delay.
- All the three rays entered the core at the same time, from the same direction. But leave the core at different times in different directions. So, the distortion is high

Graded index fibre

- In graded index optical fibre the refractive index of the core ($n_1$) is non-uniform and decreases gradually from its centre to the core-cladding boundary.

- ✪ Consider an incident ray 'AO' consists of three different wave lengths $\lambda_1$, $\lambda_2$ & $\lambda_3$. When the incident ray AO enters in to the core it is dispersed in to three rays.
- ✪ The light rays continuously bend in the core as the core refractive index gradually decreases. The motion is periodical and have same time period.




- ✪ The light rays go near to the boundary have more speed and travel more distance than the rays travel near the axis because the refractive index of the core near the boundary is less.
- ✪ All the rays enter the core at one time will leave the core at one time even though they travel different distances. So, the distortion is low.

**Fibre materials**

- ✓ Material which can be made long, thin and flexible fibres are taken.
- ✓ The material should be highly transparent that can transmit the light effectively, good chemical durability, highly resistant to electric shock.
- ✓ Majority of fibres are made of glass i.e. either Silica or Silicate.
- ✓ Addition of $GeO_2$ or $P_2O_5$ slightly increases the refractive index of $SiO_2$. (Core material)
- ✓ Addition of $B_2O_3$ slightly decreases the refractive index of $SiO_2$. (Cladding material)
- ✓ Examples of glass fibre combination

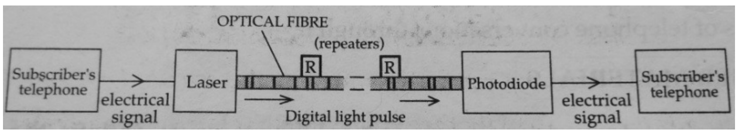
| S. No. | Core | Cladding |
|---|---|---|
| 1. | $GeO_2 – SiO_2$ | $SiO_2$ |
| 2. | $P_2O_5$ - $SiO_2$ | $SiO_2$ |
| 3. | $SiO_2$ | $B_2O_3$ - $SiO_2$ |
| 4. | $GeO_2 – B_2O_3$ - $SiO_2$ *Means $SiO_2$doped with $GeO_2$ & $B_2O_3$* | $B_2O_3$ - $SiO_2$ |

- ✓ For plastic fibres, the materials used are Teflon, Polysterine, Fluoropolymers etc. Plastic fibres are low cost. They can be used for local area net works.

**Principles of fiber communication**

- ➢ Light travels with very high speed in optical fibre than the electrical signal travels in copper cable.
- ➢ The signal strength does not fall down as the light travels by total internal reflection.
- ➢ More no. of massages can be sent in a optical fibre with clarity.

Function of each block, in the block diagram of optical communication system.




1. Subscriber's telephone :-
   - ✪ This telephone consists of a microphone. The microphone converts the sound waves in to electrical signal.

2. Laser :-
   - ✪ Electrical signals are continuous. These continuous electrical signals are converted in to digital electrical pulses by means of analog to digital converters.
   - ✪ The digital pulses are sent in to optical transmitter. The transmitter contains amplifier and semi-conductor laser. Digital optical pulses are generated in lasers.
3. Repeaters :-
   - ✪ The digital optical pulses are transmitted through the optical fibre. Amplifiers (or) boosters (or) repeaters (R) are placed at places where the pulse strength is to be amplified.
4. Photo diode :-
   - ✪ Photo diode is used to convert the digital optical pulses in to continuous electrical signal. So, photo diode is digital to analog converter (decoder).
5. Subscriber's telephone :-
   - ✪ The sound box in this telephone converts the electrical signal in to sound wave.

**Advantages of fiber optic communication**

1) Optical fibre is cheaper.
2) This communication is the fastest system than any other communication system.
3) As the optical fibres are dielectrics, there is no scope for electric shocks.
4) The energy loss per unit length is very small in this communication.
5) As they have very wide band width, too many modes of signals can be transmitted.
6) Optical fibres are flexible and they are in light weight.
7) Optical fibres are protected from interfering with electromagnetic radiation and radio frequency radiations.
8) They can with stand from extreme temperatures.
9) Since, the fibres are made of Silica the raw material availability is abundant.

# Chapter – IX

# Holography

## Introduction

The word holography originates from the Greek words "holos" (Complete) and "graphos" (Writing). It is the technique to record the complete picture of an object. This technique was proposed by a scientist named Gabor in 1947.

An ordinary photograph records the two dimensional image of the object because it records only the amplitude or the intensity distribution. But in holography technique, both the intensity as well as phase of the light wave is recorded.

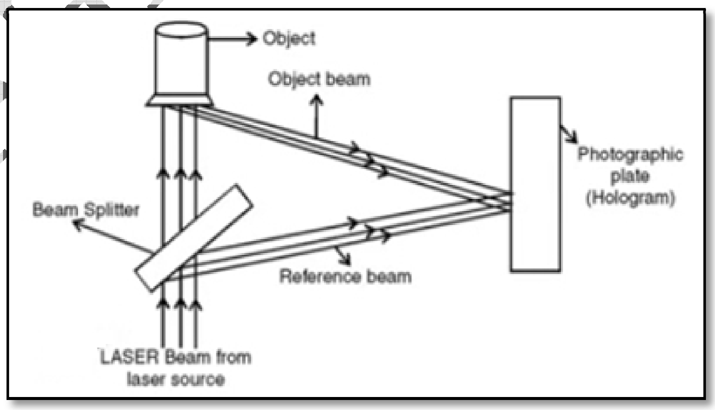
### Basic principle of holography

In the holographic plate, two beams combine and interference **pattern** will be formed. This interference **pattern** is recorded on the holographic plate. The three-dimensional image of the object can be seen by exposing the recorded holographic plate [hologram] to coherent light. This is the principle of holography.

## Gabor hologram

### Recording of hologram

The recording of hologram is based on the phenomenon of interference. It requires a LASER source, a plane mirror or beam splitter, an object and a photo graphic plate.




In holography, the light waves reflected from the object is recorded. These light waves consist of intensity and phase. The record is called hologram. The hologram has no resemblance to the original object but it contains all the information about the object in an optical code.

A laser beam from the laser source incident on a plane mirror or beam splitter. This beam splitter splits the beam in to two. One part of the splitted beam after reflection from beam spitter strikes the photographic plate. This beam is called reference beam.

While the other part of the splitted beam ( trasmitted through the splitter) strikes on the photographic plate after suffering reflections from various points of the object. This beam is called object beam.

The object beam reflected from the object interferes with the reference beam when both the beams reach the photographic plate. The superposition of these two beams produces an interference pattern (in the form of dark and bright fringes). This pattern is recorded on the photographic plate.

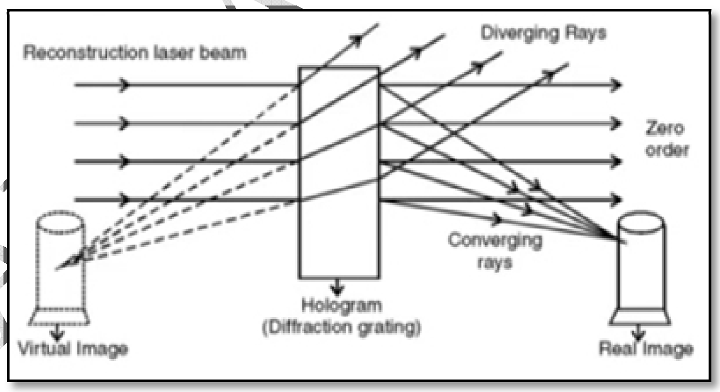
The photographic plate with recorded interference pattern is called hologram. This photographic plate is also called as Gabor zone plate in honour of Denis Gabor who developed the phenomenon of holography.

Each and every part of the hologram receives light from various points of the object. Thus, even if the hologram is broken into pieces, each piece is capable of reconstructing the whole image of the object.

**Reconstruction of the image**

It is based on the principle of diffraction. In the reconstruction process, the hologram is illuminated by laser beam and this beam is called reconstruction beam. This beam is identical to reference beam in the process of construction of hologram.

The hologram acts as a diffraction grating. The reconstruction beam will under go the phenomenon of diffraction during the passage through the hologram. The reconstruction beam. after passing through the hologram produces a real as well as virtual images of the object.




One of the diffracted beams emerging from the hologram appears to diverge from an apparent object when projected back. Thus, virtual image is formed behind the hologram at the original site of the object and the converging beam forms the real image in front of the hologram. Thus, an observer sees light waves diverging from the virtual image and the image is identical to the object.

**Limitations** :-

1) Hard to see with out good direct point of source of light.
2) Sticker hologram not visible in dark.
3) Color reproduction is not perfect.
4) It is very costly.
5) Brightness, shadows etc are difficult to adjust
6) If too many pages are stored in one crystal/photopolymer, the strength of each hologram gets diminished.
7) Needs good recording sensitive material to allow high data transfer rate.
8) Only LASER light can be used.

**Applications of holography** :-

1) Museums keep archival records in holograms.

2) Instantaneous growth rate of a live plant can be directly observed through a hologram.

3) The creation and annihilation of matter and antimatter can be seen through Bubble chamber holograms.

4) Holograms are made inside live organs through optical fibers, providing more details than any previous alternate techniques.

5) Holography is also used to detect stress in materials. A stressed material will deform. A Comparison between the before and after holograms can determine where the greatest stress is.

6) Holographic interferometry is used in laboratories for **non-destructive testing**. It visually reveals structural faults without damaging the specimen.

7) Fighter pilots use holographic displays of their instruments so they can keep looking straight up.

8) Holographic optical elements (HOE) are used to perform the functions such as lenses, mirrors, gratings, diffusers etc. They can also combine several functions together not possible with conventional optical elements.

9) Holographic scanners are used in store check-out counters for reading the bar codes.

10) HOLOSTORE is a holographic computer memory system being manufactured to replace disc drive. It will have thousands time more memory capacity and no mechanical movements.